Let all variables be integer except $L$ is a list of integers. What is the exact precondition

(a) for $x'+y' > 8$ to be refined by $x:= 1$

\[
\forall x', y'. \ x' + y' > 8 \iff (x:= 1)
\]

\[
\forall x', y'. \ x' + y' > 8 \iff x' = 1 \land y' = y
\]

\[
1 + y > 8
\]

\[
y > 7
\]

(b) for $x' = 1$ to be refined by $x:= 1$

\[
\forall x', y'. \ x' = 1 \iff (x:= 1)
\]

\[
\forall x', y'. \ x' = 1 \iff x' = 1
\]

\[
1 = 1
\]

\[
\top
\]

(c) for $x' = y$ to be refined by $y:= 1$

\[
\forall x', y'. \ x' = y \iff (y:= 1)
\]

\[
\forall x', y'. \ x' = y \iff x' = x \land y' = 1
\]

\[
x = y
\]

(d) for $x' \geq y'$ to be refined by $x:= y+z$

\[
\forall x', y', z'. \ x' \geq y' \iff (x:= y+z)
\]

\[
\forall x', y', z'. \ x' \geq y' \iff x' = y+z \land y' = y \land z' = z
\]

\[
y + z \geq y
\]

\[
z \geq 0
\]

(e) for $y' + z' \geq 0$ to be refined by $x:= y+z$

\[
\forall x', y', z'. \ y' + z' \geq 0 \iff (x:= y+z)
\]

\[
\forall x', y', z'. \ y' + z' \geq 0 \iff x' = y+z \land y' = y \land z' = z
\]

\[
y + z \geq 0
\]

(f) for $y' + z' \geq 0$ to be refined by $y:= 1$

\[
\forall x', y', z'. \ y' + z' \geq 0 \iff (y:= 1)
\]

\[
\forall x', y', z'. \ y' + z' \geq 0 \iff x' = x \land y' = 1 \land z' = z
\]

\[
y + z \geq 0
\]

(g) for $x' \leq 1 \lor x' \geq 5$ to be refined by $x:= x+1$

\[
\forall x'. \ (x' \leq 1 \lor x' \geq 5) \iff (x:= x+1)
\]

\[
\forall x'. \ (x' \leq 1 \lor x' \geq 5) \iff x' = x+1
\]

\[
x + 1 \leq 1 \lor x + 1 \geq 5
\]

\[
\neg x \ 1..4
\]

(h) for $x' < y' \land \exists x. \ L \ x < y' \to$ to be refined by $x:= 1$

\[
\forall x', y', L'. \ x' < y' \land (\exists x. \ L \ x < y') \iff (x:= 1)
\]

\[
\forall x', y', L'. \ x' < y' \land (\exists x. \ L \ x < y') \iff x' = 1 \land y' = y \land L' = L
\]

\[
1 < y \land \exists x. \ L \ x < y
\]

(i) for $\exists y. \ L \ y < x' \to$ to be refined by $x:= y+1$

\[
\forall x', y', L'. \ (\exists y. \ L \ y < x') \iff (x:= y+1)
\]

\[
\forall x', y', L'. \ (\exists y. \ L \ y < x') \iff x' = y + 1 \land y' = y \land L' = L
\]

\[
\exists x. \ L \ z < x'
\]

\[
\exists x. \ L \ z < x' + 1
\]
for $L' \ 3 = 4$ to be refined by $L:= i \rightarrow 4 \mid L$
\[
\forall L', i' \ 3 = 4 \iff (L:= i \rightarrow 4 \mid L)
\]
\[
\forall L', i' \ 3 = 4 \iff L' = i \rightarrow 4 \mid L \land i' = i
\]
\[
(i \rightarrow 4 \mid L) \ 3 = 4
\]
\[
i = 3 \lor L \ 3 = 4
\]

for $x' = a$ to be refined by if $a > b$ then $x := a$ else ok fi
\[
\forall x', a', b' \ x' = a \iff \text{if } a > b \text{ then } x := a \text{ else ok fi}
\]
\[
\text{replace if, :=, and ok}
\]
\[
\forall x', a', b' \ x' = a \iff (a > b \land x' = a \land a' = a \land b' = b) \lor (a \leq b \land x' = x \land a' = a \land b' = b)
\]
\[
\text{antidist splitting}
\]
\[
\forall x', a', b' \ x' = a \iff \text{a} \leq b \land x' = a \land a' = a \land b' = b
\]
\[
\text{specialization and identity; one-point}
\]
\[
x = a \iff a \leq b
\]

for $x' = y \land y' = x$ to be refined by $(z := x. \ x := y. \ y := z)$
\[
\forall x', y', z' \ x' = y \land y' = x \iff (z := x. \ x := y. \ y := z)
\]
\[
\text{Substitution Law}
\]
\[
\forall x', y', z' \ x' = y \land y' = x \iff (z := x. \ x := y. \ y := z)
\]
\[
\text{Substitution Law}
\]
\[
\forall x', y', z' \ x' = y \land y' = x \iff x' = y \land y' = x \land z' = y
\]
\[
\text{One-point, 3 times}
\]
\[
y = y \land x = x
\]
\[
T
\]

for $ax^2 + bx + c = 0$ to be refined by $(x := ax + b. \ x := -x/a)$
\[
\forall x' \ ax^2 + bx + c = 0 \iff (x := ax + b. \ x := -x/a)
\]
\[
\text{replace final assignment}
\]
\[
\forall x' \ ax^2 + bx + c = 0 \iff (x := ax + b. \ x' = -x/a)
\]
\[
\text{substitution law}
\]
\[
\forall x' \ ax^2 + bx + c = 0 \iff x' = -(ax + b)/a
\]
\[
\text{one point}
\]
\[
\text{This is the exact precondition.}
\]
\[
\text{But we can simplify it if we allow a sufficient precondition answer:}
\]
\[
\iff a \neq 0 \land ax^2 + bx + c = 0
\]

for $f' = n'$! to be refined by $(n := n+1. \ f := fn)$ where $n$ is natural and ! is factorial.
\[
\forall f', n' \ f' = n'! \iff (n := n+1. \ f := fn)
\]
\[
\text{expand last assignment}
\]
\[
\forall f', n' \ f' = n'! \iff (n := n+1. \ f' = fn \land n' = n)
\]
\[
\text{substitution law}
\]
\[
\forall f', n' \ f' = n'! \iff f' = fn(n+1) \land n' = n+1
\]
\[
\text{one-point twice}
\]
\[
f(n+1) = (n+1)!
\]
\[
\text{definition of !}
\]
\[
fn(n+1) = n!(n+1)
\]
\[
cancellation
\]
\[
f = n!
\]

for $7 \leq c' < 28 \land \text{odd } c'$ to be refined by $(a := b-1. \ b := a+3. \ c := a+b)$
\[
\forall a', b', c' \ 7 \leq c' < 28 \land \text{odd } c' \iff (a := b-1. \ b := a+3. \ c := a+b)
\]
\[
\text{expand last asm}
\]
\[
\forall a', b', c' \ 7 \leq c' < 28 \land \text{odd } c' \iff (a := b-1. \ b := a+3. \ a' = a \land b' = b \land c' = a+b)
\]
\[
\text{substitution law twice}
\]
\[
\forall a', b', c' \ 7 \leq c' < 28 \land \text{odd } c' \iff a' = b-1 \land b' = b+2 \land c' = 2b+1
\]
\[
\text{one-pt 3 times}
\]
\[
7 \leq 2b+1 < 28 \land \text{odd } (2b+1)
\]
\[
3 \leq b < 14
\]

for $s' = \Sigma L[0..i']$ to be refined by $(s := s + L \ i. \ i := i+1)$
\[
\forall s', i', L' \ (s' = \Sigma L[0..i']) \iff s' = s + L \ i \land i' = i+1 \land L' = L
\]
\[
s + L \ i = \Sigma L[0..i+1]
\( s = \Sigma L \) [0;..\( i \)]

(q) for \( x' > 5 \) to be refined by \( x': x + (1, 2) \)

\[ \forall x': x' > 5 \iff x': x + (1, 2) \]

\[ \forall x': x' > 5 \iff x': x + 1, x + 2 \]

\[ \forall x': x' > 5 \iff x': x + 1 \lor x': x + 2 \]

\[ \forall x': x' > 5 \iff x' = x + 1 \lor x' = x + 2 \]

\[ \forall x': (x' > 5 \iff x' = x + 1) \land (x' > 5 \iff x' = x + 2) \]

\[ (\forall x': x' > 5 \iff x' = x + 1) \land (\forall x': x' > 5 \iff x' = x + 2) \]

\[ x + 1 > 5 \land x + 2 > 5 \]

arithmetic, inclusion because \( x > 4 \Rightarrow x > 3 \)

\[ x > 4 \]

(r) for \( x' > 0 \) to be refined by \( x': x + (–1, 1) \)

\[ \forall x': x' > 0 \iff x': x + (–1, 1) \]

\[ \forall x': x' > 0 \iff x': x – 1, x + 1 \]

\[ \forall x': x' > 0 \iff x': x – 1 \lor x': x + 1 \]

\[ \forall x': x' > 0 \iff x' = x – 1 \lor x' = x + 1 \]

\[ \forall x': (x' > 0 \iff x' = x – 1) \land (x' > 0 \iff x' = x + 1) \]

\[ (\forall x': x' > 0 \iff x' = x – 1) \land (\forall x': x' > 0 \iff x' = x + 1) \]

\[ x – 1 > 0 \land x + 1 > 0 \]

arithmetic

\[ x > 1 \land x > –1 \]

inclusion because \( x > 1 \Rightarrow x > –1 \)

\[ x > 1 \]