299   Prove

(a)   the Precondition Law: A is a sufficient precondition for specification P to be refined by specification S if and only if $A \Rightarrow P$ is refined by S.

(b)   the Postcondition Law: A' is a sufficient postcondition for specification P to be refined by specification S if and only if $A' \Rightarrow P$ is refined by S.

After trying the question, scroll down to the solution.

(a)   the Precondition Law: A is a sufficient precondition for specification P to be refined by specification S if and only if $A{\Rightarrow}P$ is refined by S.

§ $(\forall\sigma\cdot\ A{\Rightarrow}(\forall\sigma'\cdot\ P{\Leftarrow}S)\ =\ \forall\sigma,\sigma'\cdot\ (A{\Rightarrow}P){\Leftarrow}S)$ distribution
$=$ $(\forall\sigma,\sigma'\cdot\ A{\Rightarrow}(P{\Leftarrow}S)\ =\ \forall\sigma,\sigma'\cdot\ (A{\Rightarrow}P){\Leftarrow}S)$ portation twice
$=$ $(\forall\sigma,\sigma'\cdot\ A{\wedge}S{\Rightarrow}P\ =\ \forall\sigma,\sigma'\cdot\ A{\wedge}S{\Rightarrow}P)$ reflexivity
$=$ $\top$

(b)   the Postcondition Law: A' is a sufficient postcondition for specification P to be refined by specification S if and only if $A'{\Rightarrow}P$ is refined by S.

§ $(\forall\sigma'\cdot\ A'{\Rightarrow}(\forall\sigma\cdot\ P{\Leftarrow}S)\ =\ \forall\sigma,\sigma'\cdot\ (A'{\Rightarrow}P){\Leftarrow}S)$ distribution
$=$ $(\forall\sigma,\sigma'\cdot\ A'{\Rightarrow}(P{\Leftarrow}S)\ =\ \forall\sigma,\sigma'\cdot\ (A'{\Rightarrow}P){\Leftarrow}S)$ portation twice
$=$ $(\forall\sigma,\sigma'\cdot\ A'{\wedge}S{\Rightarrow}P\ =\ \forall\sigma,\sigma'\cdot\ A'{\wedge}S{\Rightarrow}P)$ reflexivity
$=$ $\top$