In a language with array element assignment, the program
\[ x := i. \ i := A \ i. \ A \ i := x \]
was written with the intention to swap the values of \( i \) and \( A \ i \). Assume that all variables and array elements are of type \textit{nat}, and that \( i \) has a value that is an index of \( A \).

(a) In variables \( x, i, \) and \( A \), specify that \( i \) and \( A \ i \) should be swapped, the rest of \( A \) should be unchanged, but \( x \) might change.

\[ i' = A i \land A' = i \rightarrow i \mid A \]

(b) Find the exact precondition for which the program refines the specification of part (a).

\[ \forall x', i', A'. i' = A i \land A' = i \rightarrow i \mid A \iff (x := i. \ i := A i. \ A := i \rightarrow x \mid A) \]

expand final asmt

\[ \forall x', i', A'. i' = A i \land A' = i \rightarrow i \mid A \iff (x := i. \ i := A i. \ x' = x \land i' = i \land A' = i \rightarrow x \mid A) \]

substitution law twice

\[ \forall x', i', A'. i' = A i \land A' = i \rightarrow i \mid A \iff x' = i \land i' = A i \land A' = A \rightarrow i \mid A \]

1-pt \times 3

\[ \text{context} \iff \text{identity} \]

\[ \text{case idempotent} \]

\[ \text{context, reflexive} \]

\[ \text{One Case Law} \]

\[ \text{list equality} \]

\[ \text{split domain of} \ j \]

\[ \text{absorption} \]

\[ \text{So} \ i \ \text{and} \ A \ i \ \text{will be swapped if and only if they have the same value to start with, making the swap useless.} \]

(c) Find the exact postcondition for which the program refines the specification of part (a).

\[ \forall x, i, A. i' = A i \land A' = i \rightarrow i \mid A \iff x' = i \land i' = A i \land A' = A \rightarrow i \mid A \]

context to drop first \( i' = A i \); \( x \) doesn't appear; one-pt for \( i \); context to replace last \( A i \)

\[ \forall A. A' = x \rightarrow x' \mid A \iff i' = A x' \land A' = i' \rightarrow x' \mid A \]

case idempotent

\[ \text{context: replace} \ i' \]

\[ \text{context: replace} \ A' \]

\[ \text{specialization} \]

\[ \text{context} \iff \text{replace} \ A' \]

\[ \text{context} \iff \text{replace} \ A' \]

\[ \text{note that} \ x' \land A' \iff A' = i' \rightarrow x' \mid A \iff A' = A x' \]

\[ \text{If, in the end, we see} \ x' \land A' \iff A' = i' \rightarrow x' \mid A \iff A' = A x' \]

\[ \text{we know they were swapped (well, we won't see} \ A' \iff A' = i' \rightarrow x' \mid A \iff A' = A x' \text{because of the final assignment, so really it's just the first two possibilities).} \]