There are three people, named Front, Middle, and Back, standing in a queue. Each is wearing a hat, which is either red or blue. Back can see what color hat Middle and Front are wearing, but cannot see his own. Middle can see what color hat Front is wearing, but cannot see Back's or his own. Front cannot see what color hat anyone is wearing. A passerby tells them that at least one of them is wearing a red hat. Back then says: I don't know what color hat I'm wearing. Middle then says: I don't know what color hat I'm wearing. Front then says: I do know what color hat I'm wearing. Formalize Front's reasoning, and determine what color hat Front is wearing.

Let $b$ mean “Back is wearing red”, let $m$ mean “Middle is wearing red”, and let $f$ mean “Front is wearing red”. The passerby said

(a) $b \lor m \lor f$

Here is Front's reasoning. If Back saw that Middle and Front were wearing blue, then Back would have calculated

$$
(b \lor m \lor f) \land \neg m \land \neg f$$

$$\equiv (b \lor m \lor f) \land \neg (m \lor f)$$

so Back would have known that he is wearing a red hat. So Middle and Front conclude

(b) $m \lor f$

Now if Middle saw that Front were wearing blue, Middle would have calculated

$$
(m \lor f) \land \neg f$$

$$\Rightarrow m$$

so Middle would have known that he is wearing a red hat. So Front concludes $f$. Front is wearing red.

The puzzle could be told with more than three people, or with fewer. It can even be told about one person, but then it is just the following: One person doesn't know what color hat he is wearing. A passerby tells him it's red. So he concludes it's red. (I suppose the story can even be told about zero people: A passerby says “true”. End of story.)