278 (shift test) You are given two infinitely long lists A and B. The items can be compared for order. Both lists have period n: $nat+1$.

$$\forall k: nat\cdot\ A\ k = A\ (k+n)\ \land\ B\ k = B\ (k+n)$$

Write a program to determine if A and B are the same except for a shift of indexes.

After trying the question, scroll down to the solution.

§ The result we want is R defined as

$$R = s' = \exists i, j \cdot \forall k \cdot A(i+k)=B(j+k)$$

where quantifications are over nat, but thanks to periodicity we can take them to be over $0,..n$. Now define a to be the maximum of all segments of A of length n using list order. Define b similarly.

$$a = \Uparrow k \cdot A[k;..k+n]$$
$$b = \Uparrow k \cdot B[k;..k+n]$$

Now define Q to say that up to starting index i, all segments of A of length n are less than b, and symmetrically that up to starting index j, all segments of B of length n are less than a.

$$Q = (\forall k: 0,..i \cdot A[k;..k+n] < b) \;\land\; (\forall k: 0,..j \cdot B[k;..k+n] < a)$$

And finally, let P say that a segment of A starting at i of length h equals a segment of B starting at j of length h.

$$P = A[i;..i+h] = B[j;..j+h]$$

Now the problem is solved as follows.

$R \Leftarrow i:= 0.\ j:= 0.\ i<n \land j<n \land Q \Rightarrow R$

$i<n \land j<n \land Q \Rightarrow R \Leftarrow h:= 0.\ i<n \land j<n \land Q \land h<n \land P \Rightarrow R$

$i<n \land j<n \land Q \land h<n \land P \Rightarrow R \Leftarrow$

  if $A(i+h) < B(j+h)$ **then** $i:= i+h+1.\ j<n \land Q \Rightarrow R$

  else if $A(i+h) > B(j+h)$ **then** $j:= j+h+1.\ i<n \land Q \Rightarrow R$

   else $h:= h+1.\ i<n \land j<n \land Q \land h\le n \land P \Rightarrow R$ **fi fi**

$j<n \land Q \Rightarrow R \Leftarrow$ **if** $i\ge n$ **then** $s:= \bot$ **else** $i<n \land j<n \land Q \Rightarrow R$ **fi**

$i<n \land Q \Rightarrow R \Leftarrow$ **if** $j\ge n$ **then** $s:= \bot$ **else** $i<n \land j<n \land Q \Rightarrow R$ **fi**

$i<n \land j<n \land Q \land h\le n \land P \Rightarrow R \Leftarrow$

  if $h=n$ **then** $s:= \top$ **else** $i<n \land j<n \land Q \land h<n \land P \Rightarrow R$ **fi**

The execution time bound $3\times n$ is easily proven, but I think maybe $2\times n$ is possible.