(shift test) You are given two infinitely long lists \( A \) and \( B \). The items can be compared for order. Both lists have period \( n: \text{nat}+1 \).

\[
\forall k: \text{nat} \cdot A_k = A(k+n) \land B_k = B(k+n)
\]

Write a program to determine if \( A \) and \( B \) are the same except for a shift of indexes.

After trying the question, scroll down to the solution.
The result we want is $R$ defined as

$$R \equiv s' = \exists i, j\ \forall k\ A(i+k) = B(j+k)$$

where quantifications are over $\text{nat}$, but thanks to periodicity we can take them to be over $0..n$. Now define $a$ to be the maximum of all segments of $A$ of length $n$ using list order. Define $b$ similarly.

$$a = \uparrow k\ A[k..k+n]$$ $$b = \uparrow k\ B[k..k+n]$$

Now define $Q$ to say that up to starting index $i$, all segments of $A$ of length $n$ are less than $b$, and symmetrically that up to starting index $j$, all segments of $B$ of length $n$ are less than $a$.

$$Q \equiv (\forall k:\ 0..i\ A[k..k+n] < b) \land (\forall k:\ 0..j\ B[k..k+n] < a)$$

And finally, let $P$ say that a segment of $A$ starting at $i$ of length $h$ equals a segment of $B$ starting at $j$ of length $h$.

$$P \equiv A[i..i+h] = B[j..j+h]$$

Now the problem is solved as follows.

$$R \iff i:= 0.\ j:= 0.\ i<n \land j<n \land Q \Rightarrow R$$

$$\iff i<n \land j<n \land Q \implies h:= 0.\ i<n \land j<n \land Q \land h<n \land P \Rightarrow R$$

$$\iff i<n \land j<n \land Q \land h<n \land P \Rightarrow R$$

$$\iff \text{if } A(i+h) < B(j+h) \text{ then } i:= i+h+1.\ j<n \land Q \Rightarrow R$$

$$\iff \text{else if } A(i+h) > B(j+h) \text{ then } j:= j+h+1.\ i<n \land Q \Rightarrow R$$

$$\iff \text{else } h:= h+1.\ i<n \land j<n \land Q \land h\leq n \land P \Rightarrow R \text{ fi}$$

$$\iff j<n \land Q \Rightarrow R$$

$$\iff \text{if } j\geq n \text{ then } s:= \bot \text{ else } i<n \land j<n \land Q \Rightarrow R \text{ fi}$$

$$\iff i<n \land Q \Rightarrow R$$

$$\iff \text{if } j\geq n \text{ then } s:= \bot \text{ else } i<n \land j<n \land Q \Rightarrow R \text{ fi}$$

$$\iff i<n \land j<n \land Q \land h\leq n \land P \Rightarrow R$$

$$\iff \text{if } h=n \text{ then } s:= \top \text{ else } i<n \land j<n \land Q \land h<n \land P \Rightarrow R \text{ fi}$$

The execution time bound $3\times n$ is easily proven, but I think maybe $2\times n$ is possible.