You are given two infinitely long lists \( A \) and \( B \). The items can be compared for order. Both lists have period \( n: \text{nat} + 1 \).

\[ \forall k: \text{nat} \cdot A \ k = A \ (k+n) \land B \ k = B \ (k+n) \]

Write a program to determine if \( A \) and \( B \) are the same except for a shift of indexes.

The result we want is \( R \) defined as

\[ R = s' = \exists i, j \forall k \cdot A(i+k) = B(j+k) \]

where quantifications are over \( \text{nat} \), but thanks to periodicity we can take them to be over \( 0..n \). Now define \( a \) to be the maximum of all segments of \( A \) of length \( n \) using list order. Define \( b \) similarly.

\[ a = \uparrow k \cdot A[\ldots k+n] \]
\[ b = \uparrow k \cdot B[\ldots k+n] \]

Now define \( Q \) to say that up to starting index \( i \), all segments of \( A \) of length \( n \) are less than \( b \), and symmetrically that up to starting index \( j \), all segments of \( B \) of length \( n \) are less than \( a \).

\[ Q = (\forall k: 0..i \cdot A[\ldots k+n] < b) \land (\forall k: 0..j \cdot b[\ldots k+n] < a) \]

And finally, let \( P \) say that a segment of \( A \) starting at \( i \) of length \( h \) equals a segment of \( B \) starting at \( j \) of length \( h \).

\[ P = A[i\ldots i+h] = B[j\ldots j+h] \]

Now the problem is solved as follows.

\[ R \iff i:=0. \ j:=0. \ i<n \land j<n \land Q \Rightarrow R \]
\[ i<n \land j<n \land Q \Rightarrow R \iff h:=0. \ i<n \land j<n \land Q \land h<n \land P \Rightarrow R \]

\[ i<n \land j<n \land Q \Rightarrow R \iff \begin{cases} \text{if } A(i+h) < B(j+h) \text{ then } i:=i+h+1. \ j<n \land Q \Rightarrow R \\ \text{else if } A(i+h) > B(j+h) \text{ then } j:=j+h+1. \ i<n \land Q \Rightarrow R \\ \text{else } h:=h+1. \ i<n \land j<n \land Q \land h\leq n \land P \Rightarrow R \fi \]
\[ j<n \land Q \Rightarrow R \iff \begin{cases} \text{if } i\geq n \text{ then } s:=\bot \text{ else } i<n \land j<n \land Q \Rightarrow R \fi \]
\[ i<n \land Q \Rightarrow R \iff \begin{cases} \text{if } j\geq n \text{ then } s:=\top \text{ else } i<n \land j<n \land Q \Rightarrow R \fi \]
\[ i<n \land j<n \land Q \land h<n \land P \Rightarrow R \iff \begin{cases} \text{if } h=n \text{ then } s:=\top \text{ else } i<n \land j<n \land Q \land h<n \land P \Rightarrow R \fi \]

The execution time bound \( 3n \) is easily proven, but I think maybe \( 2n \) is possible.