(shift test) You are given two infinitely long lists $A$ and $B$. The items can be compared for order. Both lists have period $n$: $\text{nat}+1$.

$$\forall k: \text{nat}. \ A_k= A(k+n) \land B_k= B(k+n)$$

Write a program to determine if $A$ and $B$ are the same except for a shift of indexes.

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The result we want is $R$ defined as

$$R \equiv \exists i, j. \ \forall k. \ A(i+k) = B(j+k)$$

where quantifications are over $\text{nat}$, but thanks to periodicity we can take them to be over $0..n$. Now define $a$ to be the maximum of all segments of $A$ of length $n$ using list order. Define $b$ similarly.

$$a = \text{MAX} k. \ A[k;..k+n]$$
$$b = \text{MAX} k. \ B[k;..k+n]$$

Now define $Q$ to say that up to starting index $i$, all segments of $A$ of length $n$ are less than $b$, and symmetrically that up to starting index $j$, all segments of $B$ of length $n$ are less than $a$.

$$Q \equiv (\forall k: 0..i. A[k;..k+n] < b) \land (\forall k: 0..j. B[k;..k+n] < a)$$

And finally, let $P$ say that a segment of $A$ starting at $i$ of length $h$ equals a segment of $B$ starting at $j$ of length $h$.

$$P \equiv A[i;..i+h] = B[j;..j+h]$$

Now the problem is solved as follows.

$$R \iff i:=0. \ j:=0. \ i<n \land j<n \land Q \Rightarrow R$$
$$i<n \land j<n \land Q \Rightarrow R \iff h:=0. \ i<n \land j<n \land Q \land h<n \land P \Rightarrow R$$
$$i<n \land j<n \land Q \land h<n \land P \Rightarrow R \iff$$

- **if** $A(i+h) < B(j+h)$ **then** $i:= i+h+1$. $j<n \land Q \Rightarrow R$
- **else if** $A(i+h) > B(j+h)$ **then** $j:= j+h+1$. $i<n \land Q \Rightarrow R$
- **else** $h:= h+1$. $i<n \land j<n \land Q \land h\leq n \land P \Rightarrow R \text{ fi fi}$

$$j<n \land Q \Rightarrow R \iff$$

- **if** $i\geq n$ **then** $s:= \bot$ **else** $i<n \land j<n \land Q \Rightarrow R \text{ fi}$

$$i<n \land Q \Rightarrow R \iff$$

- **if** $j\geq n$ **then** $s:= \top$ **else** $i<n \land j<n \land Q \land h<n \land P \Rightarrow R \text{ fi}$

The execution time bound $3\times n$ is easily proven, but I think maybe $2\times n$ is possible.