You are given two infinitely long lists \( A \) and \( B \). The items can be compared for order. Both lists have period \( n \): \( \text{nat}+1 \).

\[
\forall k: \text{nat}. \; A(k+n) = B(k+n)
\]

Write a program to determine if \( A \) and \( B \) are the same except for a shift of indexes.

The result we want is \( R \) defined as

\[
R = s' = \exists i, j \forall k. A(i+k) = B(j+k)
\]

where quantifications are over \( \text{nat} \), but thanks to periodicity we can take them to be over \( 0..n \). Now define \( a \) to be the maximum of all segments of \( A \) of length \( n \) using list order. Define \( b \) similarly.

\[
a = \text{MAX} \; k \cdot A[k..k+n]
\]

\[
b = \text{MAX} \; k \cdot B[k..k+n]
\]

Now define \( Q \) to say that up to starting index \( i \), all segments of \( A \) of length \( n \) are less than \( b \), and symmetrically that up to starting index \( j \), all segments of \( B \) of length \( n \) are less than \( a \).

\[
Q = (\forall k: 0..i. A[k..k+n] < b) \land (\forall k: 0..j. B[k..k+n] < a)
\]

And finally, let \( P \) say that a segment of \( A \) starting at \( i \) of length \( h \) equals a segment of \( B \) starting at \( j \) of length \( h \).

\[
P = A[i..i+h] = B[j..j+h]
\]

Now the problem is solved as follows.

\[
R \iff i := 0. \; j := 0. \; i < n \land j < n \land Q \implies R
\]

\[
i < n \land j < n \land Q \implies R \iff h := 0. \; i < n \land j < n \land Q \land h < n \land P \implies R
\]

\[
i < n \land j < n \land Q \iff h := h + 1. \; i < n \land j < n \land Q \land h < n \land P \implies R \iff
\]

\[
\text{if } A(i+h) < B(j+h) \text{ then } i := i + h + 1. \; j < n \land Q \implies R
\]

\[
\text{else if } A(i+h) > B(j+h) \text{ then } j := j + h + 1. \; i < n \land Q \implies R
\]

\[
\text{else } h := h + 1. \; i < n \land j < n \land Q \land h < n \land P \implies R \iff
\]

\[
\text{if } h = n \text{ then } s := \bot \text{ else } i < n \land j < n \land Q \land h < n \land P \implies R \iff
\]

The execution time bound \( 3 \times n \) is easily proven, but I think maybe \( 2 \times n \) is possible.