270  (greatest common divisor) Write a program to find the greatest common divisor of

(a) two positive integers.
§ Let \( \text{gcd} \ a \ b \) be the greatest common divisor of \( a \) and \( b \). Then

\[
\begin{align*}
\text{if } a > b & \text{ then } a' = b' = \text{gcd} \ a \ b \land t' \leq t + a \uparrow b \\
\text{else if } a < b & \text{ then } b' = b - a. \ t := t + 1. \ a' = b' = \text{gcd} \ a \ b \land t' \leq t + a \uparrow b \\
\text{else } & \text{ ok fi fi}
\end{align*}
\]

Let all variables (except time) be of type positive natural. A program for \( \text{gcd} \) is

\[
\begin{align*}
a' = b' = \text{gcd} \ a \ b \land t' \leq t + a \uparrow b & \iff \\
\text{if } a > b & \text{ then } a := a - b. \ t := t + 1. \ a' = b' = \text{gcd} \ a \ b \land t' \leq t + a \uparrow b \\
\text{else if } a < b & \text{ then } b := b - a. \ t := t + 1. \ a' = b' = \text{gcd} \ a \ b \land t' \leq t + a \uparrow b \\
\text{else } & \text{ ok fi fi}
\end{align*}
\]

Here are more properties of \( \text{gcd} \).

\[
\begin{align*}
\text{gcd} \ (2 \times a) \ (2 \times b) & = 2 \times \text{gcd} \ a \ b \\
\text{gcd} \ (2 \times a) \ (2 \times b + 1) & = \text{gcd} \ a \ (2 \times b + 1) \\
\text{gcd} \ (2 \times a + 1) \ (2 \times b) & = \text{gcd} \ (2 \times a + 1) \ b \\
\text{gcd} \ (2 \times a + 1) \ (2 \times b + 1) & = \text{gcd} \ (2 \times a + 1) \ (b-a)
\end{align*}
\]

These properties allow us to write a \( \text{gcd} \) program that runs in \( \log (a \uparrow b) \) time.

\[
\begin{align*}
a' = b' = \text{gcd} \ a \ b \land t' \leq t + \log (a \uparrow b) & \iff \\
\text{if } a = b & \text{ then ok} \\
\text{else if even } a & \text{ then if even } b \\
& \text{ then } a := a/2. \ b := b/2. \ t := t + 1. \ a' = b' = \text{gcd} \ a \ b \land t' \leq t + \log (a \uparrow b). \\
& \text{else } a := 2 \times a. \ b := 2 \times b \\
\text{else if even } b & \text{ then } b := b/2. \ t := t + 1. \ a' = b' = \text{gcd} \ a \ b \land t' \leq t + \log (a \uparrow b) \text{ fi fi}
\end{align*}
\]

(b) two integers (not necessarily positive ones) that are not both zero.

(c) three positive integers. One method is to find the greatest common divisor of two of them, and then find the greatest common divisor of that and the remaining number, but there is a better way.