Given two natural numbers, write a program to find their quotient using only addition, subtraction, doubling, halving, test for even, and comparisons.

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I suppose the question means the natural part of the quotient, discarding the remainder. Let \( n \), \( m \), and \( q \) be natural number variables. We want \( q' = \text{div } n \ m \).

\[
q' = \text{div } n \ m \iff \begin{cases} 
  n < m & \text{then } q := 0 \\
  \text{else} & n := n - m. \quad q' = \text{div } n \ m. \quad q := q + 1
\end{cases}
\]

That solution takes time \( \text{div } n \ m \). Let's try for better.

\[
q' = \text{div } n \ m \iff \begin{cases} 
  n < m & \text{then } q := 0 \\
  \text{else if even } n & \text{then } n := n / 2. \quad q' = \text{div } n \ m. \quad q := q \times 2 \\
  \text{else} & n := n - m. \quad q' = \text{div } n \ m. \quad q := q + 1
\end{cases}
\]

That solution takes time something like \( \log n \).