Let $n$ be a natural variable. Add time according to the recursive measure, and find a finite upper bound on the execution time of

$$P \leftarrow \text{if } n \geq 2 \text{ then } n \leftarrow n-2. \quad P. \quad n \leftarrow n+1. \quad P. \quad n \leftarrow n+1 \text{ else ok fi}$$

After trying the question, scroll down to the solution.
To ensure that every loop includes a time increment, it is sufficient to put \( t := t + 1 \) just before the first call. (But the question isn’t any harder, and the time bound isn’t significantly different, if we put \( t := t + 1 \) before both calls.) Because of the two calls, each at approximately the original value of \( n \) I guess the time might be exponential. Actually, it looks just like Fibonacci: the first call is at \( n–2 \), the second is at \( n–1 \). Let’s try

\[
\begin{align*}
P & = t' \leq t + 2^n \\
\end{align*}
\]

The proof of the refinement will be by cases. First case:

\[
\begin{align*}
n \geq 2 \land (n := n–2. \ t := t+1. \ P. \ n := n+1. \ P. \ n := n+1) \\
\Rightarrow n \geq 2 \land (t' \leq t + 1 + 2^{n–2}. \ t' \leq t + 2^{n+1}. \ n' := n+1 \land t' = t) \\
\Rightarrow n \geq 2 \land \exists n'', t'', n''', t'''. \ t'' \leq t + 1 + 2^{n–2} \land t''' \leq t'' + 2^{n''+1} \land n'' = n'''+1 \land t' = t''' \\
\Rightarrow n \geq 2 \\
\end{align*}
\]

Oops. The final time seems to be completely arbitrary. The problem is that the first call of \( P \) allows \( n \) to change arbitrarily, so the last call of \( P \) allows \( t \) to change arbitrarily. Let’s try again.

\[
\begin{align*}
P & = n' = n \land t' \leq t + 2^n \\
\end{align*}
\]

The proof of the refinement will be by cases. First case:

\[
\begin{align*}
n \geq 2 \land (n := n–2. \ t := t+1. \ P. \ n := n+1. \ P. \ n := n+1) \\
\Rightarrow n \geq 2 \land n' = n \land t' \leq t + 1 + 2^{n–2} + 2^{n–1} \\
\Rightarrow n \geq 2 \land n' = n \land t' \leq t + 1 + 3 \cdot 2^{n–2} \\
\Rightarrow n' = n \land t' \leq t + 2^n \quad \text{specialize and simplify} \\
\end{align*}
\]

Last case:

\[
\begin{align*}
n < 2 \land \text{ok} \\
\Rightarrow n < 2 \land n' = n \land t' = t \\
\Rightarrow n' = n \land t' \leq t + 2^n \\
\end{align*}
\]