The question says we can double, but not multiply, so I'll take that to mean that we can multiply by 2 but not by anything else. The question says we can halve, but not divide, so I'll take that to mean that we can divide by 2 but not by anything else.

For a solution with linear time we could use
\[ a^2 = (a-1)^2 + 2 \times a - 1 \]
For a solution with logarithmic time, use
\[
\text{if even } a \text{ then } a^2 = 4 \times (a/2)^2 \text{ else } a^2 = 4 \times ((a-1)/2)^2 + 2 \times a - 1 \text{ fi}
\]
Let all variables be natural.
\[
x := a^2 \iff \text{ if } a = 0 \text{ then } x := 0
\]
\[
\text{ else if even } a \text{ then } a := a/2. \ x := a^2. \ a := a \times 2. \ x := x \times 2 \times x
\]
\[
\text{ else } a := (a-1)/2. \ x := a^2. \ a := a \times 2 + 1. \ x := x \times 2 \times 2 + ax - 1 \text{ fi}
\]
Note that in the solution, the occurrences of \( a^2 \) are recursive calls. Note also that in the usual binary representation of natural numbers, \( a \times 2 \) is just shift left, and both \( a/2 \) (for even \( a \)) and \((a-1)/2 \) (for odd \( a \)) are just shift right. The refinement can be proven in 3 cases. First case:
\[
a = 0 \land (x := 0) \implies a = 0 \land a' = a \land x' = 0 \implies a = 0 \land a' = a \land x' = a^2 \implies x := a^2
\]
Middle case:
\[
a > 0 \land \text{ even } a \land (a := a/2. \ x := a^2. \ a := a \times 2. \ x := x \times 2 \times x) \implies a > 0 \land \text{ even } a \land (a := a/2. \ x := a^2. \ a' = a \times 2 \land x' = x \times 2 \times x) \implies a > 0 \land \text{ even } a \land a' = a/2 \times 2 \land x' = (a/2)^2 \times 2 \implies a > 0 \land \text{ even } a \land a' = a \land x' = a^2 \implies x := a^2
\]
Last case:
\[
\text{odd } a \land (a := (a-1)/2. \ x := a^2. \ a := a \times 2 + 1. \ x := x \times 2 \times 2 + ax - 1) \implies \text{odd } a \land (a := (a-1)/2. \ x := a^2. \ a' = a \times 2 + 1 \land x' = x \times 4 + ax - 1) \implies \text{odd } a \land (a := (a-1)/2. \ x := a^2. \ a' = a \times 2 + 1 \land x' = x \times 4 + ax + 4) \implies \text{odd } a \land (a := (a-1)/2. \ a' = a \times 2 + 1 \land x' = (a^2) \times 4 + ax + 4) \implies \text{odd } a \land a' = a \land x' = a^2 \implies x := a^2
\]
For the timing, replace \( x := a^2 \) by \text{ if } a = 0 \text{ then } t' = t \text{ else } t' \leq t + 1 + \log a \text{ fi} \text{, and put } t := t + 1 \text{ in front of the recursive calls. The proof is by cases. First,}
\[
\text{ if } a = 0 \text{ then } t' = t \text{ else } t' \leq t + 1 + \log a \text{fi} \iff a = 0 \land x' = x \land t' = t
\]
\[
\top
\]
The second case, right side, is
\[
a \neq 0 \land \text{ even } a \land (a := a/2. \ t := t + 1.
\text{ if } a = 0 \text{ then } t' = t \text{ else } t' \leq t + 1 + \log a \text{ fi.}
\]
\[
a := a \times 2. \ x := x \times 2 \times x)
\]
\[
\begin{align*}
\Rightarrow & \quad a \neq 0 \land \text{even } a \land \text{if } a/2 = 0 \text{ then } t' = t + 1 \text{ else } t' \leq t + 2 + \log (a/2) \text{ fi} \\
\Rightarrow & \quad a \neq 0 \land \text{even } a \land t' \leq t + 2 + \log (a/2) \\
\Rightarrow & \quad a \neq 0 \land \text{even } a \land t' \leq t + 1 + \log a \\
\text{which is the left side. The third case, right side, is} \quad a \neq 0 \land \text{odd } a \land \text{if } (a-1)/2 = 0 \text{ then } t' = t + 1 \text{ else } t' \leq t + 2 + \log ((a-1)/2) \text{ fi} \\
\Rightarrow & \quad a \neq 0 \land \text{odd } a \land \text{if } a = 1 \text{ then } t' = t + 1 \text{ else } t' \leq t + 1 + \log (a-1) \text{ fi} \\
\text{which is the left side.} \\
\end{align*}
\]

Here's the best solution. Define

\[ P \equiv y' = y + x \cdot n \land \text{if } x = 0 \text{ then } t' = t \text{ else } t' \leq t + \log x \text{ fi} \]

Then the program is

\[
\begin{align*}
\text{if } x > 0 \Rightarrow P & \iff \text{if even } x \text{ then even } x \Rightarrow P \text{ else odd } x \Rightarrow P \text{ fi} \\
\text{odd } x \Rightarrow P & \iff \text{if } x = 0 \text{ then } \text{ok} \text{ else even } x \land x > 0 \Rightarrow P \text{ fi} \\
\text{even } x \land x > 0 \Rightarrow P & \iff n := 2 \cdot x \cdot n. x := x/2. t := t + 1. x > 0 \Rightarrow P \\
x > 0 \Rightarrow P & \iff \text{if even } x \text{ then even } x \land x > 0 \Rightarrow P \text{ else odd } x \Rightarrow P \text{ fi} \\
\end{align*}
\]