23 Let p and q be binary expressions. Suppose p is both a theorem and an antitheorem (the theory is inconsistent).

(a) Prove, using the rules of proof presented, that q is both a theorem and an antitheorem.

(b) Is $q=q$ a theorem or an antitheorem?

After trying the question, scroll down to the solution.

(a) Prove, using the rules of proof presented, that q is both a theorem and an antitheorem.

§

	q	identity law
$=$	$q \vee \bot$	since p is an antitheorem, replace $\bot$ by p
$=$	$q \vee p$	since p is a theorem, replace p by $\top$
$=$	$q \vee \top$	base law
$=$	$\top$	

so q is a theorem.

	q	identity law
$=$	$q \wedge \top$	since p is a theorem, replace $\top$ by p
$=$	$q \wedge p$	since p is an antitheorem, replace p by $\bot$
$=$	$q \wedge \bot$	base law
$=$	$\bot$	

so q is an antitheorem.

(b) Is $q=q$ a theorem or an antitheorem?

§ In part (a), q was any binary expression. So it could be $q=q$. So $q=q$ is both a theorem and an antitheorem. Or, if you prefer, in part (a) q is a binary variable, so by the Instance Rule, every binary expression is both a theorem and an antitheorem, and in particular, $q=q$ is both a theorem and an antitheorem.