Let $p$ and $q$ be binary expressions. Suppose $p$ is both a theorem and an antitheorem (the theory is inconsistent).

(a) Prove, using the rules of proof presented, that $q$ is both a theorem and an antitheorem.

(b) Is $q=q$ a theorem or an antitheorem?

After trying the question, scroll down to the solution.
(a) Prove, using the rules of proof presented, that \( q \) is both a theorem and an antitheorem.

\[
\begin{align*}
q & \quad \text{identity law} \\
\equiv q \lor \bot & \quad \text{since } p \text{ is an antitheorem, replace } \bot \text{ by } p \\
\equiv q \lor p & \quad \text{base law} \\
\equiv q \lor \top & \quad \text{since } p \text{ is a theorem, replace } p \text{ by } \top \\
\equiv \top & \quad \text{base law} \\
\end{align*}
\]

so \( q \) is a theorem.

\[
\begin{align*}
q & \quad \text{identity law} \\
\equiv q \land \top & \quad \text{since } p \text{ is a theorem, replace } \top \text{ by } p \\
\equiv q \land p & \quad \text{base law} \\
\equiv q \land \bot & \quad \text{since } p \text{ is an antitheorem, replace } p \text{ by } \bot \\
\equiv \bot & \quad \text{base law} \\
\end{align*}
\]

so \( q \) is an antitheorem.

(b) Is \( q=q \) a theorem or an antitheorem?

\[
\begin{align*}
q & \quad \text{identity law} \\
\equiv q \land \top & \quad \text{since } p \text{ is a theorem, replace } \top \text{ by } p \\
\equiv q \land p & \quad \text{base law} \\
\equiv q \land \bot & \quad \text{since } p \text{ is an antitheorem, replace } p \text{ by } \bot \\
\equiv \bot & \quad \text{base law} \\
\end{align*}
\]

so \( q \) is an antitheorem.