226 (minimum sum segment) Given a list of integers, possibly including negatives, write a program to find

(a)√ the minimum sum of any nonempty segment (sublist of consecutive items).

(b) the nonempty segment whose sum is minimum.

(c) the minimum sum of any segment, including empty segments.

After trying the question, scroll down to the solution.

(a)√ the minimum sum of any nonempty segment (sublist of consecutive items).

§ As it says in the textbook Subsection 5.2.3, we define

$P = s' = ⇓i: 0,..\#L· ⇓j: i+1,..\#L+1· Σ L [i;..j]$

$A\ k = s=(⇓i: 0,..k· ⇓j: i+1,..k+1· Σ L [i;..j]) ∧ c=(⇓i: 0,..k· Σ L [i;..k])$

Then

$P ⇐ s:= ∞.\ c:= ∞.\ A\ 0 ⇒ A'(\#L)$

$A\ 0 ⇒ A'(\#L) ⇐$ **for** $k:= 0;..\#L$ **do** $k: 0,..\#L ∧ A\ k ⇒ A'(k+1)$ **od**

$k: 0,..\#L ∧ A\ k ⇒ A'(k+1) ⇐ c:= c↓0 + L\ k.\ s:= s↓c$

Proof: First refinement:

$s:= ∞.\ c:= ∞.\ A\ 0 ⇒ A'(\#L)$ replace $A\ 0$ and $A'(\#L)$

$=$ $s:= ∞.\ c:= ∞.$
$s=(⇓i: 0,..0· ⇓j: i+1,..1· Σ L [i;..j]) ∧ c=(⇓i: 0,..0· Σ L [i;..0])$
$⇒ s'=(⇓i: 0,..\#L· ⇓j: i+1,..\#L+1· Σ L [i;..j]) ∧ c'=(⇓i: 0,..\#L· Σ L [i;..\#L])$
minimum of an empty bunch is $∞$

$=$ $s:= ∞.\ c:= ∞.\ s=∞ ∧ c=∞ ⇒ P ∧ c'=(⇓i: 0,..\#L· Σ L [i;..\#L])$
substitution law twice; P does not mention s or c

$=$ $∞=∞ ∧ ∞=∞ ⇒ P ∧ c'=(⇓i: 0,..\#L· Σ L [i;..\#L])$

$⇒$ P

Middle refinement: no proof needed.

Last refinement:

$k: 0,..\#L ∧ A\ k ⇒ A'(k+1)$ expand $A\ k$ and $A'(k+1)$

$=$ $k: 0,..\#L ∧ s=(⇓i: 0,..k· ⇓j: i+1,..k+1· Σ L [i;..j]) ∧ c=(⇓i: 0,..k· Σ L [i;..k])$
$⇒ s'=(⇓i: 0,..k+1· ⇓j: i+1,..k+2· Σ L [i;..j]) ∧ c'=(⇓i: 0,..k+1· Σ L [i;..k])$ context

$=$ $k: 0,..\#L ∧ s=(⇓i: 0,..k· ⇓j: i+1,..k+1· Σ L [i;..j]) ∧ c=(⇓i: 0,..k· Σ L [i;..k])$
$⇒ s'= s↓(c↓0 + L\ k) ∧ c'= c↓0 + L\ k$

$⇐$ $c' = c↓0 + L\ k ∧ s' = s↓(c↓0 + L\ k)$

$=$ $c:= c↓0 + L\ k.\ s:= s↓c$

(b) the nonempty segment whose sum is minimum.

§ Let $m;..n$ be the nonempty segment ending at or before k whose sum is minimum (corresponding to s), and let $h;..k$ be the nonempty segment ending at k whose sum is minimum (corresponding to c). Then

$s:= ∞.\ c:= ∞.$
for $k:= 0;..\#L$
do if $c≤0$ **then** $c:= c + L\ k$ **else** $c:= L\ k.\ h:= k$ **fi**.
if $s≤c$ **then** *ok* **else** $s:= c.\ m:= h.\ n:= k$ **fi od**

(c) the minimum sum of any segment, including empty segments.

§ To allow empty segments, we define

$P = s' = ⇓i: 0,..\#L+1· ⇓j: i,..\#L+1· Σ L [i;..j]$

$A\ k = s=(⇓i: 0,..k+1· ⇓j: i,..k+1· Σ L [i;..j]) ∧ c=(⇓i: 0,..k+1· Σ L [i;..k])$

Then

$P ⇐ s:= 0.\ c:= 0.\ A\ 0 ⇒ A'(\#L)$

$A\ 0 ⇒ A'(\#L) ⇐$ **for** $k:= 0;..\#L$ **do** $k: 0,..\#L ∧ A\ k ⇒ A'(k+1)$ **od**

$k: 0,..\#L ∧ A\ k ⇒ A'(k+1) ⇐ c:= (c + L\ k)↓0.\ s:= s↓c$

The first and last refinements need proof; the middle refinement does not need proof.