201 (duplicate count) Write a program to find how many items are duplicates (repeats) of earlier items

(a) in a given sorted nonempty list.

(b) in a given list.

After trying the question, scroll down to the solution.

(a) in a given sorted nonempty list.

§ Let the list be L . Let n and j be natural state variables. The result will be reported as n' . Let's call the specification S , defined as

$$S \;=\; n' = ¢(§i\colon 1,..\#L\cdot L\,i = L(i-1)) \;\wedge\; t' = t+\#L-1$$

Also define

$$P \;=\; 1{\le}j{\le}\#L \;\wedge\; n' = n + ¢(§i\colon j,..\#L\cdot L\,i = L(i-1)) \;\wedge\; t' = t+\#L-j$$

The problem is solved by the refinements

$$S \;\Leftarrow\; n{:=}\,0.\;\; j{:=}\,1.\;\; P$$

$$P \;\Leftarrow\; \textbf{if}\ j{=}\#L\ \textbf{then}\ ok\ \textbf{else}\ \textbf{if}\ L\,j = L(j-1)\ \textbf{then}\ n{:=}\,n{+}1\ \textbf{else}\ ok\ \textbf{fi}.\;\; j{:=}\,j{+}1.\;\; t{:=}\,t{+}1.\;\; P\ \textbf{fi}$$

Proof: NOT YET WRITTEN

(b) in a given list.

Maybe the best way is to check if the list is nonempty, sort it, then use the solution of part (a). Here is another way. If item $L\,k$ is a duplicate of earlier item $L\,i$, then $L\,i$ is a duplicate of later item $L\,k$. So it is equivalent to write a program to find how many items are duplicates (repeats) of later items.

Define:

$$S \;=\; n' = ¢(§j\colon 0,..\#L\cdot \exists m\colon j{+}1,..\#L\cdot L\,j = L\,m) \;\wedge\; t' \le t+(\#L)^2/2$$

$$P \;=\; n' = n + ¢(§j\colon i,..\#L\cdot \exists m\colon j{+}1,..\#L\cdot L\,j = L\,m) \;\wedge\; t' \le t+(\#L-i)^2/2$$

$$Q \;=\; n' = n + \textbf{if}\ \exists m\colon k,..\#L\cdot L\,i = L\,m\ \textbf{then}\ 1\ \textbf{else}\ 0\ \textbf{fi} + ¢(§j\colon i{+}1,..\#L\cdot \exists m\colon j{+}1,..\#L\cdot L\,j = L\,m) \;\wedge\; t' \le t+(\#L-k)+(\#L-(i+1))^2/2$$

Refine:

$$S \;\Leftarrow\; n{:=}\,0.\;\; i{:=}\,0.\;\; P$$

$$P \;\Leftarrow\; \textbf{if}\ i{=}\#L\ \textbf{then}\ ok\ \textbf{else}\ k{:=}\,i{+}1.\;\; t{:=}\,t{+}1.\;\; Q\ \textbf{fi}$$

$$Q \;\Leftarrow\; \textbf{if}\ k{=}\#L\ \textbf{then}\ i{:=}\,i{+}1.\;\; P\ \textbf{else}\ \textbf{if}\ L\,i = L\,k\ \textbf{then}\ n{:=}\,n{+}1.\;\; i{:=}\,i{+}1.\;\; P\ \textbf{else}\ k{:=}\,k{+}1.\;\; t{:=}\,t{+}1.\;\; Q\ \textbf{fi}$$

Prove:

$n{:=}\,0.\;\; i{:=}\,0.\;\; P$ — expand P

$=\;\; n{:=}\,0.\;\; i{:=}\,0.\;\; n' = n + ¢(§j\colon i,..\#L\cdot \exists m\colon j{+}1,..\#L\cdot L\,j = L\,m) \;\wedge\; t' \le t+(\#L-i)^2/2$ — substitution law twice

$=\;\; n' = 0 + ¢(§j\colon 0,..\#L\cdot \exists m\colon j{+}1,..\#L\cdot L\,j = L\,m) \;\wedge\; t' \le t+(\#L-0)^2/2$ — arithmetic

$=\;\; n' = ¢(§j\colon 0,..\#L\cdot \exists m\colon j{+}1,..\#L\cdot L\,j = L\,m) \;\wedge\; t' \le t+(\#L)^2/2$ — contract S

$=\;\; S$

$i{=}\#L \wedge ok$ — expand ok

$=\;\; i{=}\#L \wedge n'{=}n \wedge i'{=}i \wedge k'{=}k \wedge t'{=}t$

$\Rightarrow\;\; n' = n + ¢(§j\colon i,..\#L\cdot \exists m\colon j{+}1,..\#L\cdot L\,j = L\,m) \;\wedge\; t' \le t+(\#L-i)^2/2$ — contract P

$=\;\; P$

$i{\ne}\#L \wedge (k{:=}\,i{+}1.\;\; Q)$ — expand Q

$=\;\; i{\ne}\#L \wedge (k{:=}\,i{+}1.\;\; t{:=}\,t{+}1.\;\; n' = n + \textbf{if}\ \exists m\colon k,..\#L\cdot L\,i = L\,m\ \textbf{then}\ 1\ \textbf{else}\ 0\ \textbf{fi} + ¢(§j\colon i{+}1,..\#L\cdot \exists m\colon j{+}1,..\#L\cdot L\,j = L\,m) \;\wedge\; t' \le t+(\#L-k)+(\#L-(i+1))^2/2)$ — substitution law twice

$=\quad i \neq \#L \;\wedge\; n' = n + \textbf{if } \exists m\colon i{+}1,..\#L\cdot L\,i = L\,m \textbf{ then } 1 \textbf{ else } 0 \textbf{ fi}$
$\qquad\qquad + ¢(§j\colon i{+}1,..\#L\cdot \exists m\colon j{+}1,..\#L\cdot L\,j = L\,m)$
$\qquad \wedge\; t' \le t{+}1{+}(\#L{-}(i{+}1)){+}(\#L{-}(i{+}1))^2/2$

$=\quad i \neq \#L \;\wedge\; n' = n + ¢(§j\colon i\cdot \exists m\colon j{+}1,..\#L\cdot L\,j = L\,m)$
$\qquad\qquad + ¢(§j\colon i{+}1,..\#L\cdot \exists m\colon j{+}1,..\#L\cdot L\,j = L\,m)$
$\qquad \wedge\; t' \le t{+}(\#L{-}i)^2/2 + 1/2$

combine the two ¢ and time is *xnat* so lose the 1/2 and drop $i \neq \#L$

$\Rightarrow\quad n' = n + ¢(§j\colon i,..\#L\cdot \exists m\colon j{+}1,..\#L\cdot L\,j = L\,m) \;\wedge\; t' \le t{+}(\#L{-}i)^2/2$ contract P

$=\quad P$

$k{=}\#L \wedge (i{:=}\, i{+}1.\;\; P)$ expand P

$=\quad k{=}\#L \wedge (i{:=}\, i{+}1.$
$\qquad n' = n + ¢(§j\colon i,..\#L\cdot \exists m\colon j{+}1,..\#L\cdot L\,j = L\,m) \;\wedge\; t' \le t{+}(\#L{-}i)^2/2)$

substitution law

$=\quad k{=}\#L \;\wedge\; n' = n + ¢(§j\colon i{+}1,..\#L\cdot \exists m\colon j{+}1,..\#L\cdot L\,j = L\,m) \;\wedge\; t' \le t{+}(\#L{-}(i{+}1))^2/2$

add 0 to n and to time

$=\quad k{=}\#L$
$\wedge\; n' = n + \textbf{if } \bot \textbf{ then } 1 \textbf{ else } 0 \textbf{ fi}$
$\qquad + ¢(§j\colon i{+}1,..\#L\cdot \exists m\colon j{+}1,..\#L\cdot L\,j = L\,m)$
$\wedge\; t' \le t{+}(\#L{-}k){+}(\#L{-}(i{+}1))^2/2$

using context $k{=}\#L$ expand $\bot$

$=\quad k{=}\#L$
$\wedge\; n' = n + \textbf{if } \exists m\colon k,..\#L\cdot L\,i = L\,m \textbf{ then } 1 \textbf{ else } 0 \textbf{ fi}$
$\qquad + ¢(§j\colon i{+}1,..\#L\cdot \exists m\colon j{+}1,..\#L\cdot L\,j = L\,m)$
$\wedge\; t' \le t{+}(\#L{-}k){+}(\#L{-}(i{+}1))^2/2$ drop $k{=}\#L$

$\Rightarrow\quad n' = n + \textbf{if } \exists m\colon k,..\#L\cdot L\,i = L\,m \textbf{ then } 1 \textbf{ else } 0 \textbf{ fi}$
$\qquad + ¢(§j\colon i{+}1,..\#L\cdot \exists m\colon j{+}1,..\#L\cdot L\,j = L\,m)$
$\wedge\; t' \le t{+}(\#L{-}k){+}(\#L{-}(i{+}1))^2/2$ contract Q

$=\quad Q$

$k \neq \#L \;\wedge\; L\,i = L\,k \;\wedge\; (n{:=}\, n{+}1.\;\; i{:=}\, i{+}1.\;\; P)$ expand P

$=\quad k \neq \#L \;\wedge\; L\,i = L\,k$
$\wedge\; (n{:=}\, n{+}1.\;\; i{:=}\, i{+}1.$
$\qquad n' = n + ¢(§j\colon i,..\#L\cdot \exists m\colon j{+}1,..\#L\cdot L\,j = L\,m) \;\wedge\; t' \le t{+}(\#L{-}i)^2/2)$

substitution law twice

$=\quad k \neq \#L \;\wedge\; L\,i = L\,k$
$\wedge\; n' = n + 1 + ¢(§j\colon i{+}1,..\#L\cdot \exists m\colon j{+}1,..\#L\cdot L\,j = L\,m)$
$\wedge\; t' \le t{+}(\#L{-}(i{+}1))^2/2$ STEPS AND JUSTIFICATIONS NEEDED HERE

$\Rightarrow\quad n' = n + \textbf{if } \exists m\colon k,..\#L\cdot L\,i = L\,m \textbf{ then } 1 \textbf{ else } 0 \textbf{ fi}$
$\qquad + ¢(§j\colon i{+}1,..\#L\cdot \exists m\colon j{+}1,..\#L\cdot L\,j = L\,m)$
$\wedge\; t' \le t{+}(\#L{-}k){+}(\#L{-}(i{+}1))^2/2$ contract Q

$=\quad Q$

$k \neq \#L \;\wedge\; L\,i \neq L\,k \;\wedge\; (k{:=}\, k{+}1.\;\; t{:=}\, t{+}1.\;\; Q)$ expand Q

$=\quad k \neq \#L \;\wedge\; L\,i \neq L\,k$
$\wedge\; (k{:=}\, k{+}1.\;\; t{:=}\, t{+}1.$
$\qquad n' = n + \textbf{if } \exists m\colon k,..\#L\cdot L\,i = L\,m \textbf{ then } 1 \textbf{ else } 0 \textbf{ fi}$
$\qquad\quad + ¢(§j\colon i{+}1,..\#L\cdot \exists m\colon j{+}1,..\#L\cdot L\,j = L\,m)$
$\qquad \wedge\; t' \le t{+}(\#L{-}k){+}(\#L{-}(i{+}1))^2/2)$ substitution law twice

$=\quad k \neq \#L \;\wedge\; L\,i \neq L\,k$
$\wedge\; n' = n + \textbf{if } \exists m\colon k{+}1,..\#L\cdot L\,i = L\,m \textbf{ then } 1 \textbf{ else } 0 \textbf{ fi}$
$\qquad + ¢(§j\colon i{+}1,..\#L\cdot \exists m\colon j{+}1,..\#L\cdot L\,j = L\,m)$

$\wedge\ \ t' \le t{+}1{+}(\#L{-}(k{+}1)){+}(\#L{-}(i{+}1))^2/2)$

MORE STEPS AND JUSTIFICATIONS NEEDED HERE

$\Longrightarrow\quad n' = n + \textbf{if}\ \exists m{:}\ k,..\#L\cdot\ L\ i = L\ m\ \textbf{then}\ 1\ \textbf{else}\ 0\ \textbf{fi}$

$\quad + ¢(\S j{:}\ i{+}1,..\#L\cdot\ \exists m{:}\ j{+}1,..\#L\cdot\ L\ j = L\ m)$

$\wedge\ \ t' \le t{+}(\#L{-}k){+}(\#L{-}(i{+}1))^2/2$ contract Q

$=\quad Q$