(missing number) You are given an unsorted list of length \( n \) whose items are the numbers 0..\( n+1 \) with one number missing. Write a program to find the missing number.

After trying the question, scroll down to the solution.
Let the given list be $L$ (a constant), and its length is $n$ (a constant). Then

$$L(0..n): 0..n+1 \land \forall i, j: 0..n \cdot i+j \Rightarrow L i \neq L j$$

Let $m: \text{nat}$ be a variable whose final value will be the missing number. The problem can be stated

$$m': 0..n+1 \land \neg m': L(0..n)$$

One way to solve the problem is with an extra list variable $M: [(n+1)*\text{bin}]$ to record which numbers are present in $L$. Here's an easier way: the problem is

$$m' = (\Sigma[0..n+1]) - (\Sigma L)$$

and its solution is

$$m' = (\Sigma[0..n+1]) - (\Sigma L) \iff m := n \times (n+1)/2. \quad A 0 \Rightarrow A'n$$

$$A 0 \Rightarrow A'n \iff \text{for } i := 0..n \text{ do } i: 0..n \land A i \Rightarrow A'(i+1) \text{ od}$$

$$i: 0..n \land A i \Rightarrow A'(i+1) \iff m := m - L i$$

where invariant $A i \equiv m = (\Sigma[0..n+1]) - (\Sigma L[0..i])$.

Similarly the problem can be stated as

$$m' = n + \Sigma i: 0..n \cdot i - L i$$

and solved as

$$m' = n + \Sigma i: 0..n \cdot i - L i \iff m := n. \quad B 0 \Rightarrow B'n$$

$$B 0 \Rightarrow B'n \iff \text{for } j := 0..n \text{ do } j: 0..n \land B j \Rightarrow B'(j+1) \text{ od}$$

$$j: 0..n \land B j \Rightarrow B'(j+1) \iff m := m + j - L j$$

where invariant $B j \equiv m = n + \Sigma i: 0..n \cdot i - L i$. For either solution, recursive time is $i' = t+n$. 
