198 (missing number) You are given an unsorted list of length n whose items are the numbers $0,..n+1$ with one number missing. Write a program to find the missing number.

After trying the question, scroll down to the solution.

§ Let the given list be L (a constant), and its length is n (a constant). Then

$$L(0,..n)\colon 0,..n+1 \;\wedge\; \forall i, j\colon 0,..n\cdot\ i \neq j \Rightarrow L\,i \neq L\,j$$

Let $m\colon nat$ be a variable whose final value will be the missing number. The problem can be stated

$$m'\colon 0,..n+1 \;\wedge\; \neg\, m'\colon L(0,..n)$$

One way to solve the problem is with an extra list variable $M\colon [(n+1)*bin]$ to record which numbers are present in L. Here's an easier way: the problem is

$$m' = (\Sigma[0;..n+1]) - (\Sigma L)$$

and its solution is

$$m' = (\Sigma[0;..n+1]) - (\Sigma L) \;\Longleftarrow\; m{:=}\ n{\times}(n+1)/2.\ A\,0 \Rightarrow A'n$$

$$A\,0 \Rightarrow A'n \;\Longleftarrow\; \mathbf{for}\ i{:=}\ 0;..n\ \mathbf{do}\ i\colon 0,..n \wedge A\,i \Rightarrow A'(i+1)\ \mathbf{od}$$

$$i\colon 0,..n \wedge A\,i \Rightarrow A'(i+1) \;\Longleftarrow\; m{:=}\ m - L\,i$$

where invariant $A\,i \;=\; m = (\Sigma[0;..n+1]) - (\Sigma L[0;..i])$.

Similarly the problem can be stated as

$$m' = n + \Sigma i\colon 0,..n\cdot\ i - L\,i$$

and solved as

$$m' = n + \Sigma i\colon 0,..n\cdot\ i - L\,i \;\Longleftarrow\; m{:=}\ n.\ B\,0 \Rightarrow B'n$$

$$B\,0 \Rightarrow B'n \;\Longleftarrow\; \mathbf{for}\ j{:=}\ 0;..n\ \mathbf{do}\ j\colon 0,..n \wedge B\,j \Rightarrow B'(j+1)\ \mathbf{od}$$

$$j\colon 0,..n \wedge B\,j \Rightarrow B'(j+1) \;\Longleftarrow\; m{:=}\ m + j - L\,j$$

where invariant $B\,j \;=\; m = n + \Sigma i\colon 0,..j\cdot\ i - L\,i$. For either solution, recursive time is $t' = t+n$.