Write a program to count the number of occurrences of a given item in a given 2-dimensional array in which each row is sorted and each column is sorted. The execution time must be linear in the sum of the dimensions.

Let $A$ be the array, let its size be $m \times n$, and let $x$ be the item (these are constants). The solution will use variables $r, a, b, c : \mathbb{N}$. Variable $r$ will indicate a row; on that row there may be some occurrences of item $x$; $a$ will indicate the start of these occurrences, and $b$ will indicate the end of these occurrences. Variable $c$ will accumulate the count of the occurrences of $x$. As $r$ decreases from $m$ to 0, variables $a$ and $b$ will trace the envelope of the occurrences of $x$, as in the following picture.

\[
\begin{array}{c}
\text{The problem is } P, \text{ defined as } \\
P = c' = \Sigma i : 0..m \Sigma j : 0..n \text{ if } A i j = x \text{ then } 1 \text{ else } 0 \text{ fi} \\
\text{Define } \\
Q = c = (\Sigma i : r..m \Sigma j : 0..n \text{ if } A i j = x \text{ then } 1 \text{ else } 0 \text{ fi}) \\
\quad \land \quad (\forall i : 0..r \forall j : 0..a \cdot A i j < x) \land (\forall i : r..m \forall j : b..n \cdot x < A i j) \\
\Rightarrow P \\
L = (\forall j : 0..a \cdot A r j < x) \\
\Rightarrow (\forall j : 0..a' \cdot A r j < x) \land (A r a' \geq x \land a' = n) \land r' = r \land b' = b \land c' = c \\
R = (\forall j : 0..b \cdot A r j \leq x) \\
\Rightarrow (\forall j : 0..b' \cdot A r j \leq x) \land (A r b' > x \land b' = n) \land r' = r \land a' = a \land c' = c
\end{array}
\]

Or, alternatively, define
\[
Q = (a = 0 \lor A r (a - 1) < x) \land (b = 0 \lor A r (b - 1) \leq x) \\
\Rightarrow c' = c + (\Sigma i : 0..r \Sigma j : 0..n \text{ if } A i j = x \text{ then } 1 \text{ else } 0 \text{ fi}) \\
L = (a' = 0 \lor A r (a' - 1) < x) \land (A r a' \geq x \land a' = n) \land r' = r \land b' = b \land c' = c \\
R = (b' = 0 \lor A r (b' - 1) \leq x) \land (A r b' > x \land b' = n) \land r' = r \land a' = a \land c' = c
\]

Now the refinements:
\[
P \leftarrow r' = m. \quad a := 0. \quad b := 0. \quad c := 0. \quad Q \\
Q \leftarrow \text{if } r = 0 \text{ then ok else } r := r - 1. \quad L. \quad R. \quad c := c + b - a. \quad Q \text{ fi} \\
L \leftarrow \text{if } a = n \text{ then ok else if } A r a \geq x \text{ then ok else } a := a + 1. \quad L \text{ fi} \\
R \leftarrow \text{if } b = n \text{ then ok else if } A r b > x \text{ then ok else } b := b + 1. \quad R \text{ fi} \quad \text{fi}
\]

For the time, put $t := t + 1$ before each of the three recursive calls, replace specification $P$ with $t' \leq t + m + 2n$, and replace all the other specifications $Q$, $L$, and $R$ with $a \leq n \land b \leq n \Rightarrow t' \leq t + r + m + 2n - a - b$. 

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\begin{array}{c}
\end{array}
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