ternary search) The problem is similar to binary search (Exercise 181). The strategy this time is to identify which third of the list contains the item if it occurs at all, then which ninth, then which twenty-seventh, and so on.

Let \( L \) be the list, let \( x \) be the value sought, let \( p \) be a binary variable whose final value says whether \( x \) is present in \( L \), and let \( h \), \( i \), \( j \), \( k \) be indexes. The problem is defined as

\[
x: L(0,..\#L) = p' \implies Lh'=x
\]

In general, the indexes will be in the order \( 0 \leq h \leq i \leq j \leq k \leq \#L \) and the segment still to be searched is \( h;..k \) and indexes \( i \) and \( j \) are at one-third and two-thirds of the way through the segment. So define \( R \) as

\[
R = (x: L(h;..k) = p' \implies Lh'=x)
\]

Solve the problem as follows.

\[
(x: L(0,..\#L) = p' \implies Lh'=x) \iff h:=0. \ k:=\#L. \ h<k \Rightarrow R
\]

\[
h<k \Rightarrow R \iff \begin{cases} \text{if } k-h = 1 \text{ then } p:= Lh=x \\ \text{else if } k-h = 2 \text{ then if } Lh=x \text{ then } p:= \top \ \text{else } h:= h+1. \ p:= Lh=x \fi \\ \text{else } k-h \geq 3 \Rightarrow R \fi
\]

\[
k-h \geq 3 \Rightarrow R \iff \begin{cases} i:= \text{div} (h+k) 3. \ j:= \text{div} (h + 2\times k) 3. \\ \text{if } L i > x \text{ then } k:= i \ \text{else if } L j \leq x \text{ then } h:= j \ \text{else } h:= i. \ k:= j \fi \fi \fi
\]

Again, the proofs are not very different from Subsection 4.2.5. The timing, let \( lbt \) be the logarithm base 3 function.

\[
t' \leq t + \text{ceil} \ lbt (\#L) \iff h:=0. \ k:=\#L. \ h<k \Rightarrow t' \leq t + \text{ceil} \ lbt (k-h)
\]

\[
h<k \Rightarrow t' \leq t + \text{ceil} \ lbt (k-h) \iff \begin{cases} \text{if } k-h = 1 \text{ then } p:= Lh=x \\ \text{else if } k-h = 2 \text{ then if } Lh=x \text{ then } p:= \top \ \text{else } h:= h+1. \ p:= Lh=x \fi \\ \text{else } k-h \geq 3 \Rightarrow t' \leq t + \text{ceil} \ lbt (k-h) \fi
\]

\[
k-h \geq 3 \Rightarrow t' \leq t + \text{ceil} \ lbt (k-h) \iff \begin{cases} i:= \text{div} (h+k) 3. \ j:= \text{div} (h + 2\times k) 3. \\ \text{if } L i > x \text{ then } k:= i \ \text{else if } L j \leq x \text{ then } h:= j \ \text{else } h:= i. \ k:= j \fi \fi \fi
\]

Again, the proofs are not very different from Subsection 4.2.5.