Pascal's triangle) Given \( n: \text{nat} \) and variable \( P: [*[\text{nat}]] \), write a program to assign to \( P \) a Pascal's triangle of size \( n \). For example, if \( n = 4 \), then

\[
P' = [ [1];
[1; 1];
[1; 2; 1];
[1; 3; 3; 1] ]
\]

The left side and diagonal are all 1s; each interior item is the sum of the item above it and the item diagonally above and left.

§
This solution uses some notations from Chapter 5, although they are not necessary. Also, I need \( n \) to be a variable.

\[
A \iff\begin{array}{ll}
\text{if } n=0 \text{ then } P := \text{[nil]} & \\
\text{else if } n=1 \text{ then } P := \text{[[1]]} & \\
\text{else } n := n-1. & A. \ n := n+1. \ B \fi
\end{array}
\]

\[
B \iff P := P :: [\text{[n*1]}]. \ \text{for } i := 1;..n-1 \text{ do } P n i := P (n-1) (i-1) + P (n-1) i \ \text{od}
\]

\[
A \equiv P'=(\text{Pascal's triangle of size } n) \land n'=n
\]

\[
B \equiv n\geq2 \land P=(\text{Pascal's triangle of size } n-1) \Rightarrow A
\]

Specifications \( A \) and \( B \) are partly informal, and an informal proof is easy and convincing. But it isn't hard to formalize completely.

\[
A \equiv \#
\begin{array}{ll}
P'=n'=n
\land \forall i: 0..n' \#(P'i)=i+1 \land P'i 0=1=P'i i \land \forall j: 1..i' P'i j\ =\ P'(i-1)(j-1) + P'(i-1)j
\end{array}
\]

\[
B \equiv \begin{array}{ll}
( n\geq2 \land \#P=n-1
\land \forall i: 0..n-1 \cdot #(P i)=i+1 \land P i 0=1=P i i \land \forall j: 1..i' P i j = P(i-1)(j-1) + P(i-1)j
\Rightarrow A
\end{array}
\]