182 (Pascal's triangle) Given n: *nat* and variable P: [*[*nat*]] , write a program to assign to P a Pascal's triangle of size n . For example, if $n = 4$, then

$$P' = [\ [1];\\ [1;\ 1];\\ [1;\ 2;\ 1];\\ [1;\ 3;\ 3;\ 1]\ ]$$

The left side and diagonal are all 1s; each interior item is the sum of the item above it and the item diagonally above and left.

After trying the question, scroll down to the solution.

§ This solution uses some notations from Chapter 5, although they are not necessary. Also, I need n to be a variable.

$$A \Leftarrow \textbf{if } n=0 \textbf{ then } P:= [nil]$$
$$\textbf{else if } n=1 \textbf{ then } P:= [[1]]$$
$$\textbf{else } n:= n-1.\ A.\ n:= n+1.\ B \textbf{ fi fi}$$

$$B \Leftarrow P:= P\ ;;\ [[n*1]].\ \textbf{for } i:= 1;..n-1 \textbf{ do } P\ n\ i:= P\ (n-1)\ (i-1) + P\ (n-1)\ i \textbf{ od}$$

$$A = P'=(\text{Pascal's triangle of size } n) \wedge n'=n$$
$$B = n\ge 2 \wedge P=(\text{Pascal's triangle of size } n-1) \Rightarrow A$$

Specifications A and B are partly informal, and an informal proof is easy and convincing. But it isn't hard to formalize completely.

$$A = \#P'=n'=n$$
$$\wedge\ \forall i: 0,..n\cdot \#(P'i)=i+1 \wedge P'i\ 0=1=P'i\ i \wedge \forall j: 1,..i\cdot P'i\ j = P'(i-1)(j-1) + P'(i-1)j$$

$$B = (\ n\ge 2 \wedge \#P=n-1$$
$$\wedge\ \forall i: 0,..n-1\cdot \#(P\ i)=i+1 \wedge P\ i\ 0=1=P\ i\ i \wedge \forall j: 1,..i\cdot P\ i\ j = P(i-1)(j-1) + P(i-1)j$$
$$\Rightarrow A\ )$$