

178 (polynomial) You are given $n: nat$, $c: n*rat$, $x: rat$ and variable $y: rat$. c is a string of coefficients of a polynomial (“of degree $n-1$ ”) to be evaluated at x . Write a program for

$$y' = \sum_{i: 0..n} c_i \times x^i$$

After trying the question, scroll down to the solution.

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$$\begin{aligned}
 y' &= (\sum_{i: 0..n} c_i \times x^i) \wedge t' = t+n \iff \\
 y &:= 0. \quad k := 0. \quad y' = y + (\sum_{i: k..n} c_i \times x^i) \wedge t' = t+n-k \\
 y' &= y + (\sum_{i: k..n} c_i \times x^i) \wedge t' = t+n-k \iff \\
 \text{if } k &= n \text{ then } ok \\
 \text{else } y &:= y + c_k \times x^k. \quad k := k+1. \quad t := t+1. \quad y' = y + (\sum_{i: k..n} c_i \times x^i) \wedge t' = t+n-k \text{ fi}
 \end{aligned}$$

A more efficient way (in real time), not using exponentiation, (and with less floating-point roundoff error, if you're stuck with floating-point,) is Horner's Rule, as follows:

$$\begin{aligned}
 y' &= (\sum_{i: 0..n} c_i \times x^i) \wedge t' = t+n \iff \\
 y &:= 0. \quad k := n. \quad y' = y \times x^k + (\sum_{i: 0..k} c_i \times x^i) \wedge t' = t+k \\
 y' &= y \times x^k + (\sum_{i: 0..k} c_i \times x^i) \wedge t' = t+k \iff \\
 \text{if } k &= 0 \text{ then } ok \\
 \text{else } k &:= k-1. \quad y := y \times x + c_k. \quad t := t+1. \quad y' = y \times x^k + (\sum_{i: 0..k} c_i \times x^i) \wedge t' = t+k \text{ fi}
 \end{aligned}$$