(Pascal's triangle) Given \( n: \text{nat} \) and variable \( P: [*[*\text{nat}]] \), write a program to assign to \( P \) a Pascal's triangle of size \( n \). For example, if \( n = 4 \), then
\[
P' = \begin{bmatrix}
1; 1; \\
1; 2; 1; \\
1; 3; 3; 1
\end{bmatrix}
\]
The left side and diagonal are all 1s; each interior item is the sum of the item above it and the item diagonally above and left.

§

This solution uses some notations from Chapter 5, although they are not necessary. Also, I need \( n \) to be a variable.

\[
A \iff \text{if } n=0 \text{ then } P := \text{[nil]} \text{ else if } n=1 \text{ then } P := [[1]] \text{ else } n := n-1. \quad A. \quad n := n+1. \quad B \quad \text{fi fi}
\]

\[
B \iff P := P ;; [[n*1]]. \quad \text{for } i := 1;..n-1 \text{ do } P n i := P (n-1) (i-1) + P (n-1) i \od
\]

\[
A \equiv P'=(\text{Pascal's triangle of size } n) \land n'=n
\]

\[
B \equiv n\geq2 \land P=(\text{Pascal's triangle of size } n-1) \Rightarrow A
\]

Specifications \( A \) and \( B \) are partly informal, and an informal proof is easy and convincing. But it isn't hard to formalize completely.

\[
A \equiv \#P'=n'=n
\]
\[\land \forall i: 0..n: \#(P'i)=i+1 \land P' i 0=1= P'i i \land \forall j: 1..i: P'i j = P'(i-1)(j-1) + P'(i-1)j
\]

\[
B \equiv \begin{cases} n\geq2 \land \#P=n-1 \\
\land \forall i: 0..n-1: \#(Pi)=i+1 \land Pi 0=1=Pi i \land \forall j: 1..i: Pi j = P(i-1)(j-1) + P(i-1)j
\end{cases} \Rightarrow A
\]