Given a specification $P$ and a prestate $\sigma$ with $t$ as time variable, we might define “the exact precondition for termination” as follows:

\[
\exists n : \text{nat} \cdot \forall \sigma' : t' \leq t+n \iff P
\]

Letting $x$ be an integer variable, find the exact precondition for termination of the following, and comment on whether it is reasonable.

(a)  
\[
x \geq 0 \Rightarrow t' \leq t+x
\]

\[\exists n : \text{nat} : \forall x', t' \leq t+n \iff (x \geq 0 \Rightarrow t' \leq t+x) \quad \text{idempotence for } x' ; \text{ material imp}
\]

\[\exists n : \text{nat} : \forall t' : t' \leq t+n \iff (x < 0 \vee t' \leq t+x) \quad \text{antidistribution; distribution}
\]

\[\exists n : \text{nat} : (\forall t' : t' \leq t+n \iff x < 0) \land (\forall t' : t' \leq t+n \iff t' \leq t+x) \quad \text{distrib; connection}
\]

\[\exists n : \text{nat} : t = \infty \iff x < 0 \land t+n \geq t+x
\]

If $x$ starts with a nonnegative value, it seems reasonable that $x \geq 0 \Rightarrow t' \leq t+x$ requires termination. If the computation starts at time $\infty$ (which means it never starts, because it comes after an infinite loop), it seems unreasonable to require termination. On the other hand, you cannot observe a computation starting at time $\infty$ and then failing to terminate, so this disjunct is vacuous.

(b)  
\[
\exists n : \text{nat} : t' \leq t+n
\]

\[\exists m : \text{nat} : \forall x', t' \leq t+m \iff \exists n : \text{nat} : t' \leq t+n
\]

\[\exists m : \forall x' : t' \leq t+m \iff t' \leq t+n
\]

\[\exists m : \forall n : n \leq m \vee t = \infty
\]

If the computation starts at a finite time, this specification does not require termination. Although the specification says that there is a constant upper bound on the execution time, it leaves that constant totally unspecified. An observer would never be able to complain that a computation has taken too long — even if the computation is infinite. But if the computation starts at time $\infty$, the comment of part (a) applies.

(c)  
\[
\exists f : \text{int} \rightarrow \text{nat} : t' \leq t + f\cdot x
\]

\[\exists m : \forall x' : t' \leq t+m \iff \exists f : \text{int} \rightarrow \text{nat} : t' \leq t + f\cdot x
\]

\[\exists m : \forall f : t' \leq t+m \iff t' \leq t + f\cdot x
\]

\[\exists m : \forall f : f\cdot x \leq m \vee t = \infty
\]

The paper **Halting According to aPToP** is based on the formula for the exact precondition for termination.