166 Let n be a natural variable, and let b be a binary variable. Write a program to determine whether 3 is a factor of n (whether 3 divides evenly into n with no remainder), reporting the answer as the final value of b. Your program can use addition, subtraction, comparison, and binary operators, but not multiplication, division, *div*, *mod*, *floor*, or *ceil*. (Your non-program specifications can use anything.)

(a) Write a formal specification.

(b) Refine your specification to obtain a program.

After trying the question, scroll down to the solution.

(a) Write a formal specification.

§ $b'=(mod\ n\ 3 = 0)$

(b) Refine your specification to obtain a program.

§ $b'=(mod\ n\ 3 = 0) \Leftarrow$ **if** $n<3$ **then** $b:= n=0$
else $n:= n-3.\ \ b'=(mod\ n\ 3 = 0)$ **fi**

Proof by cases: **then**-case:

	$n<3 \land (b:= n=0)$	expand assignment
$=$	$n<3 \land b'=(n=0) \land n'=n$	if $n<3$ then $mod\ n\ 3 = n$
$=$	$n<3 \land b'=(mod\ n\ 3 = 0) \land n'=n$	specialization
$\Rightarrow$	$b'=(mod\ n\ 3 = 0)$	

else-case:

	$n\ge3 \land (n:= n-3.\ \ b'=(mod\ n\ 3 = 0))$	substitution law
$=$	$n\ge3 \land b'=(mod\ (n-3)\ 3 = 0)$	if $n\ge3$ then $mod\ (n-3)\ 3 = mod\ n\ 3$
$=$	$n\ge3 \land b'=(mod\ n\ 3 = 0)$	specialization
$\Rightarrow$	$b'=(mod\ n\ 3 = 0)$	