There are 256 operators with 3 binary operands and a binary result. How many of them are degenerate? An operator is degenerate if its result can be expressed without using all the operands.

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Let $D_n$ be the number of degenerate binary operators with $n$ operands. Let $G_n$ be the number of nondegenerate (good) binary operators with $n$ operands.

$$G_n + D_n = 2^{2^n}$$

To begin with, there are 2 operators with 0 operands, namely $\top$ and $\bot$, and both of them are good. So

$G_0 = 2$ and $D_0 = 0$

Next, there are 4 operators with 1 operand, and 2 of them are good (identity and negation), and two of them are degenerate (the two constant functions). So

$G_1 = 2$ and $D_1 = 2$

Next, there are 16 operators with 2 operands. But there are 2 good operators that use 0 of the operands (that don't use either operand), and 2 good operators that use only the left operand, and 2 good operators that use only the right operand. So that's 6 operators that don't use all (both) the operands.

$G_2 = 10$ and $D_2 = 6$

More generally, the number of degenerate operators with $n$ operands is the number of good operators for each combination of fewer than $n$ operands. Let $C_{n\,m}$ be the number of ways of choosing $m$ things from among $n$ things. Let $n!$ be the factorial function.

$$n! = \prod i: 0..n\ i+1 = 1\times2\times3\times...\times n$$

$$C_{n\,m} = \frac{n!}{(m!\times(n-m)!)})$$

$$D_n = \sum i: 0..n\ C_{n\,i}\times G_{i} = Cn0\times G0 + Cn1\times G1 + ... + Cn(n-1)\times G(n-1)$$

We want $D_3$, which is

$$D_3 = C\ 3\ 0 \times G\ 0 + C\ 3\ 1 \times G\ 1 + C\ 3\ 2 \times G\ 2$$

$$= 1\times2 + 3\times2 + 3\times10$$

$$= 38$$

Therefore 38 of the operators with 3 binary operands and a binary result are degenerate.