Is the refinement

\[
P \iff \begin{cases} 
  \text{if } x = 0 \text{ then } \text{ok} \text{ else } \\ 
  x := x - 1. \ t := t + 1. \ P
\end{cases}
\]

a theorem when

\[P = x < 0 \Rightarrow x' = 1 \land t' = \infty\]

Is this reasonable? Explain.

§ Yes, it is a theorem. Proof by cases:

\[
(x < 0 \Rightarrow x' = 1 \land t' = \infty) \iff x = 0 \land \text{ok}
\]

\[
= x' = 1 \land t' = \infty \iff x < 0 \land x = 0 \land \text{ok}
\]

\[
= x' = 1 \land t' = \infty \iff \bot
\]

\[
= T
\]

\[
(x < 0 \Rightarrow x' = 1 \land t' = \infty) \iff x \neq 0 \land (x := x - 1. \ t := t + 1. \ x < 0 \Rightarrow x' = 1 \land t' = \infty)
\]

\[
= x' = 1 \land t' = \infty \iff x < 0 \land (x - 1 < 0 \Rightarrow x' = 1 \land t' = \infty)
\]

\[
= x' = 1 \land t' = \infty \iff x < 0 \land x' = 1 \land t' = \infty
\]

\[
= T
\]

When \(x < 0\) the execution time is infinite (\(t' = \infty\)) so there is no final state. It is therefore somewhat unreasonable to say \(x' = 1\). On the other hand, no observation can ever show otherwise.