Let $x$ and $n$ be natural variables. Find a specification $P$ such that both the following hold:

$$x = x' \times 2^n' \iff n := 0. \quad P$$

$$P \iff \text{if even } x \text{ then } x := x/2. \quad n := n+1. \quad P \text{ else ok fi}$$

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$$P \models x = x' \times 2^{n'-n}$$

Proof:

$n := 0. \quad x = x' \times 2^{n'-n}$

$$\models x = x' \times 2^n'$$

The second one is proven by cases. First,

$$\text{even } x \land (x := x/2. \quad n := n+1. \quad x = x' \times 2^{n'-n})$$

use Substitution Law twice

$$\models \text{even } x \land x/2 = x' \times 2^{n'-\left(n+1\right)}$$

number theory

$$\models \text{even } x \land x = x' \times 2^{n'-n}$$

specialization

$$\Rightarrow P$$

Now the other case:

$$(x = x' \times 2^{n'-n} \iff \neg \text{even } x \land \text{ok})$$

expand $\text{ok}$

$$\models (x = x' \times 2^{n'-n} \iff \neg \text{even } x \land x' = x \land n' = n)$$

assume antecedent in consequent

$$\models (x = x \times 2^{n-n} \iff \neg \text{even } x \land x' = x \land n' = n)$$

number theory

$$(\text{T} \iff \neg \text{even } x \land x' = x \land n' = n)$$

base law for $\iff$

$$\models \text{T}$$