Is the refinement

\[ P \iff \begin{cases} \text{if } x = 0 \text{ then } \text{ok} \text{ else } x := x - 1. \ t := t + 1. \ P \end{cases} \]

a theorem when

\[ P \equiv x < 0 \Rightarrow x' = 1 \land t' = \infty \]

Is this reasonable? Explain.

§ Yes, it is a theorem. Proof by cases:

\[
\begin{align*}
(x < 0 \Rightarrow x' = 1 \land t' = \infty) \iff x = 0 \land \text{ok} & \quad \text{portation} \\
x' = 1 \land t' = \infty \iff x < 0 \land x = 0 \land \text{ok} & \\
x' = 1 \land t' = \infty \iff \bot & \\
T & \\
(x < 0 \Rightarrow x' = 1 \land t' = \infty) \iff x \neq 0 \land (x := x - 1. \ t := t + 1. \ x < 0 \Rightarrow x' = 1 \land t' = \infty) & \quad \text{portation and two substitutions} \\
x' = 1 \land t' = \infty \iff x < 0 \land (x - 1 < 0 \Rightarrow x' = 1 \land t' = \infty) & \quad \text{discharge} \\
x' = 1 \land t' = \infty \iff x < 0 \land x' = 1 \land t' = \infty & \quad \text{specialization} \\
T & 
\end{align*}
\]

When \( x < 0 \) the execution time is infinite (\( t' = \infty \)) so there is no final state. It is therefore somewhat unreasonable to say \( x' = 1 \). On the other hand, no observation can ever show otherwise.