146 Let n be a natural variable. Here is a refinement.

$$P \Leftarrow \textbf{if } n=0 \textbf{ then } ok \textbf{ else } n:= n-1.\ P.\ n:= n+1 \textbf{ fi}$$

(a) Ignoring time, prove this refinement where

$$P = ok$$

(b) Now add recursive time and prove this refinement where

$$P = t:= t+n$$

After trying the question, scroll down to the solution.

(a) Ignoring time, prove this refinement where

$P \Leftarrow ok$

§	**if** n=0 **then** ok **else** n:= n–1. P. n:= n+1 **fi**	definition of P
$\Leftarrow$	**if** n=0 **then** ok **else** n:= n–1. ok. n:= n+1 **fi**	expand final assignment
$=$	**if** n=0 **then** ok **else** n:= n–1. ok. $n' = n+1$ **fi**	ok is identity for .
$=$	**if** n=0 **then** ok **else** n:= n–1. $n' = n+1$ **fi**	substitution law
$=$	**if** n=0 **then** ok **else** $n' = n-1+1$ **fi**	simplify
$=$	**if** n=0 **then** ok **else** $n'=n$ **fi**	definition of ok
$=$	**if** n=0 **then** ok **else** ok **fi**	case idempotent
$=$	ok	definition of P
$=$	P	

(b) Now add recursive time and prove this refinement where

$P \Leftarrow t{:=}\ t+n$

§	**if** n=0 **then** ok **else** n:= n–1. t:= t+1. P. n:= n+1 **fi**	definition of P
$=$	**if** n=0 **then** ok **else** n:= n–1. t:= t+1. t:= t+n. n:= n+1 **fi**	expand final assignment
$=$	**if** n=0 **then** ok **else** n:= n–1. t:= t+1. t:= t+n. $n' = n+1 \land t'=t$ **fi**	substitution law
$=$	**if** n=0 **then** ok **else** n:= n–1. t:= t+1. $n' = n+1 \land t'=t+n$ **fi**	substitution law
$=$	**if** n=0 **then** ok **else** n:= n–1. $n' = n+1 \land t'=t+1+n$ **fi**	substitution law
$=$	**if** n=0 **then** ok **else** $n' = n-1+1 \land t'=t+1+n-1$ **fi**	simplify
$=$	**if** n=0 **then** ok **else** $n'=n \land t'=t+n$ **fi**	definition of ok
$=$	**if** n=0 **then** $n'=n \land t'=t$ **else** $n'=n \land t'=t+n$ **fi**	context
$=$	**if** n=0 **then** $n'=n \land t'=t+n$ **else** $n'=n \land t'=t+n$ **fi**	case idempotent
$=$	$n'=n \land t'=t+n$	definition of assignment
$=$	$t{:=}t+n$	definition of P
$=$	P	