

139 In natural variables  $n$  and  $m$  prove  
 $P \Leftarrow n := n+1. \text{ if } n=10 \text{ then } ok \text{ else } m := m-1. \text{ P fi}$   
where  $P = m := m+n-9. \quad n := 10.$

After trying the question, scroll down to the solution.

$\S \quad n := n+1. \text{ if } n=10 \text{ then } ok \text{ else } m := m-1. P \text{ fi}$  replace *ok* and *P*  
 $= \quad n := n+1. \text{ if } n=10 \text{ then } m' = m \wedge n' = n \text{ else } m := m-1. m := m+n-9. n := 10 \text{ fi}$  replace final assignment  
 $= \quad n := n+1. \text{ if } n=10 \text{ then } m' = m \wedge n' = n \text{ else } m := m-1. m := m+n-9. m' = m \wedge n' = 10 \text{ fi}$  two substitutions  
 $= \quad n := n+1. \text{ if } n=10 \text{ then } m' = m \wedge n' = n \text{ else } m' = m-1+n-9 \wedge n' = 10 \text{ fi}$  substitution  
 $= \quad \text{if } n+1=10 \text{ then } m' = m \wedge n' = n+1 \text{ else } m' = m-1+n+1-9 \wedge n' = 10 \text{ fi}$  arithmetic  
 $= \quad \text{if } n=9 \text{ then } m' = m \wedge n' = n+1 \text{ else } m' = m+n-9 \wedge n' = 10 \text{ fi}$  context  
 $= \quad \text{if } n=9 \text{ then } m' = m+n-9 \wedge n' = 10 \text{ else } m' = m+n-9 \wedge n' = 10 \text{ fi}$  case idempotent  
 $= \quad m' = m+n-9 \wedge n' = 10$  definitions of assignment and sequential composition  
 $= \quad m := m+n-9. n := 10$   
 $= \quad P$