138 (factorial) In natural variables n and f prove

$$f{:=}\ n! \ \Leftarrow\ \textbf{if}\ n{=}0\ \textbf{then}\ f{:=}\ 1\ \textbf{else}\ n{:=}\ n{-}1.\ f{:=}\ n!.\ n{:=}\ n{+}1.\ f{:=}\ f{\times}n\ \textbf{fi}$$

where $n! = 1{\times}2{\times}3{\times}...{\times}n$.

After trying the question, scroll down to the solution.

§ Proof by cases.

	$n=0 \land (f:= 1)$	expand assignment
$=$	$n=0 \land f'=1 \land n'=n$	use context from left conjunct to change middle conjunct
$=$	$n=0 \land f'=n! \land n'=n$	contract assignment
$=$	$n=0 \land (f:=n!)$	specialization
$\Rightarrow$	$f:= n!$	

	$n\ne0 \land (n:= n-1.\ f:= n!.\ n:= n+1.\ f:= f\times n)$	
		specialize (drop conjunct) and expand last assignment
$\Rightarrow$	$n:= n-1.\ f:= n!.\ n:= n+1.\ f'=f\times n \land n'=n$	substitution
$=$	$n:= n-1.\ f:= n!.\ f'=f\times(n+1) \land n'=n+1$	substitution
$=$	$n:= n-1.\ f'= n!\times(n+1) \land n'=n+1$	simplify
$=$	$n:= n-1.\ f'= (n+1)! \land n'=n+1$	substitution
$=$	$f'= n! \land n'=n$	definition of assignment
$=$	$f:= n!$	