133 We have Refinement by Steps, Refinement by Parts, and Refinement by Cases. In this question we propose Refinement by Alternatives:

If $A \Leftarrow$ **if** b **then** C **else** D **fi** and $E \Leftarrow$ **if** b **then** F **else** G **fi** are theorems,
then $A \vee E \Leftarrow$ **if** b **then** $C \vee F$ **else** $D \vee G$ **fi** is a theorem.

If $A \Leftarrow B.C$ and $D \Leftarrow E.F$ are theorems, then $A \vee D \Leftarrow B \vee E.\ C \vee F$ is a theorem.

If $A \Leftarrow B$ and $C \Leftarrow D$ are theorems, then $A \vee C \Leftarrow B \vee D$ is a theorem.

Discuss the merits and demerits of this proposed law.

After trying the question, scroll down to the solution.

§ A law has to be a theorem, and this proposed law isn't. As a counterexample to the sequential composition part of the law, take

$$A = B = C = x'=x$$
$$D = x'=x+2$$
$$E = F = x'=x+1$$

Then $A \Leftarrow B.C$ and $D \Leftarrow E.F$ are theorems, but $A \lor D \Leftarrow B \lor E.\ C \lor F$ is not a theorem. But the other two parts of the proposed law are theorems, so refinements that use only **if**s can be composed as disjunctions just the way Refinement by Parts allows us to compose refinements as conjunctions. On the other hand, a disjunction can always be refined by just refining one of the disjuncts, so I guess this law wouldn't be very useful even if it were a theorem.