Let \( x \) be an integer state variable, and there are no other state variables.

(a) For what exact precondition does \( x := x^2 \) make \( x \) be even?

§ (the exact precondition for \( \text{even } x' \) to be refined by \( x := x^2 \))

\[
\begin{align*}
= \quad & \forall x'. \text{even } x' \iff (x := x^2) \\
= \quad & \forall x'. \text{even } x' \iff x' = x^2 \\
= \quad & \text{even } x^2 \\
= \quad & \text{even } x
\end{align*}
\]

(b) What does it mean to say that your answer to part (a) is the exact precondition for \( x := x^2 \) to make \( x \) be even?

§ It means that squaring \( x \) makes \( x \) be even if and only if \( x \) was even already.

(c) For what exact postcondition does \( x := x^2 \) make \( x \) be even?

§ (the exact postcondition for \( \text{even } x' \) to be refined by \( x := x^2 \))

\[
\begin{align*}
= \quad & \forall x'. \text{even } x' \iff (x := x^2) \\
= \quad & \forall x'. \text{even } x' \iff x' = x^2 \\
= \quad & \text{even } x' \iff \exists x. x' = x^2 \\
= \quad & \text{even } x' \iff x': \text{int}^2 \\
= \quad & \text{even } x' \lor \neg x': \text{int}^2
\end{align*}
\]

(d) What does it mean to say that your answer to part (c) is the exact postcondition for \( x := x^2 \) to make \( x \) be even?

§ It means that squaring \( x \) makes \( x \) be even if and only the final value of \( x \) is even or the final value if \( x \) is not a square. Well, the final value of \( x \) will be a square, so I guess that just means squaring \( x \) makes \( x \) even if and only if the final value of \( x \) is even, which is not very helpful.