For what exact precondition and postcondition does the following assignment move integer variable $x$ farther from zero?

(a) $x := x + 1$

§ (the exact precondition for $abs x' > abs x$ to be refined by $x := x + 1$)

\[
\begin{align*}
&\forall x': abs x' > abs x \iff (x := x + 1) \\
&\forall x': abs x' > abs x \iff x' = x + 1 & \text{one-point} \\
&abs (x + 1) > abs x \\
&x \geq 0 \\
&\text{(the exact postcondition for $abs x' > abs x$ to be refined by $x := x + 1$)}
\end{align*}
\]

§
\[
\begin{align*}
&\forall x: abs x' > abs x \iff (x := x + 1) \\
&\forall x: abs x' > abs x \iff x' = x + 1 & \text{one-point} \\
&abs (x + 1) > abs x \\
&x \geq 0 \\
&\text{(the exact postcondition for $abs x' > abs x$ to be refined by $x := x + 1$)}
\end{align*}
\]

(b) $x := abs (x + 1)$

§ (the exact precondition for $abs x' > abs x$ to be refined by $x := abs (x + 1)$)

\[
\begin{align*}
&\forall x': abs x' > abs x \iff (x := abs (x + 1)) \\
&\forall x': abs x' > abs x \iff x' = abs (x + 1) & \text{divide domain} \\
&\forall x: nat \cdot abs x' > abs x \iff x' = abs (x + 1) \\
&\land (\forall x: nat \cdot abs x' > abs x \iff x' = abs (x + 1)) & \text{change variable} \\
&\land (\forall x: nat \cdot abs x' > abs x \iff x' = abs (x + 1)) & \text{remove abs twice} \\
&\land (\forall z: nat \cdot abs x' > abs (\cdot z - 1) \iff x' = abs (\cdot z - 1 + 1)) & \text{simplify and remove abs} \\
&\land (\forall z: nat \cdot abs x' > x \iff x' = x + 1) & \text{context} \\
&\land (\forall z: nat \cdot abs x' > z + 1 \iff x' = z) & \text{context} \\
&\land (\forall z: nat \cdot abs (x + 1) > x \iff x' = x + 1) & \text{remove abs and simplify} \\
&\land (\forall z: nat \cdot abs z > z + 1 \iff x' = z) & \text{remove abs and simplify} \\
&\land (\forall x: nat \cdot T \iff x' = x + 1) & \\
&\land (\forall z: nat \cdot T \iff x' = z) \\
&\land T \land (\forall z: nat \cdot x' + z) \\
&x' < 0
\end{align*}
\]

(c)$^\vee$

$x := x^2$

§ (the exact precondition for $abs x' > abs x$ to be refined by $x := x^2$)

\[
\begin{align*}
&\forall x: int \cdot abs x' > abs x \iff x' = x^2 & \text{arithmetic: } x^2 = (\cdot x)^2 \\
&\forall x: int \cdot abs x' > abs x \iff x' = (abs x)^2 & \text{change variable} \\
&\forall y: int \cdot abs x' > y \iff x' = y^2 & \text{context} \\
&\forall y: nat \cdot abs (\cdot y^2) > y \iff x' = y^2 & \text{arithmetic: } y^2 \geq 0 \\
&\forall y: nat \cdot y^2 > y \iff x' = y^2 & \text{domain split} \\
&\land (\forall y: 0 \cdot y^2 > y \iff x' = y^2) \land (\forall y: 1 \cdot y^2 > y \iff x' = y^2) & \text{one-point} \\
&\land (\forall y: nat\cdot+2 \cdot y^2 > y \iff x' = y^2) & \text{and arithmetic} \\
&\land (\bot \iff x' = 0) \land (\bot \iff x' = 1) \land (\forall y: nat\cdot+2 \cdot T \iff x' = y^2) \\
&x' + 0 \land x' + 1
\end{align*}
\]