131 For which kinds of specifications P and Q is the following a theorem:

(a) $\neg(P.\neg Q) \Leftarrow P.Q$

(b) $P.Q \Leftarrow \neg(P.\neg Q)$

(c) $P.Q = \neg(P.\neg Q)$

After trying the question, scroll down to the solution.

§ First, rewrite the two sides.

$$\neg(P.\neg Q) \;=\; \forall\sigma''\cdot\langle\sigma'\cdot P\rangle\sigma'' \Rightarrow \langle\sigma\cdot Q\rangle\sigma''$$

$$P.Q \;=\; \exists\sigma''\cdot\langle\sigma'\cdot P\rangle\sigma'' \wedge \langle\sigma\cdot Q\rangle\sigma''$$

(a) $\neg(P.\neg Q) \;\Leftarrow\; P.Q$

§ If, for all prestates, P is deterministic, then (a) is a theorem. (That's sufficient, but not necessary.)

(b) $P.Q \;\Leftarrow\; \neg(P.\neg Q)$

§ If, for all prestates, P is satisfiable (P is implementable), then (b) is a theorem.

(c) $P.Q \;=\; \neg(P.\neg Q)$

§ If, for all prestates, P is satisfiable and deterministic, then (c) is a theorem.