Let all variables be integer except $L$ is a list of integers. What is the exact precondition for

(a) $x' + y' > 8$ to be refined by $x := 1$

(b) $x' = 1$ to be refined by $x := 1$

(c) $x' = 2$ to be refined by $x := 1$

(d) $x' = y$ to be refined by $y := 1$

== $\forall x', y' . x' = y \iff (y := 1)$

== $\forall x', y'. x' = y \iff x' = x \land y' = 1$

== $x = y$

== one-point twice

(e) $x' \geq y'$ to be refined by $x := y + z$

== $z \geq 0$

== $y' + z' \geq 0$ to be refined by $x := y + z$

== $\forall x', y', z'. y' + z' \geq 0 \iff (x := y + z)$

== $\forall x', y', z'. y' + z' \geq 0 \iff x' = y + z \land y' = y \land z' = z$

== $y + z \geq 0$

== One-point, 3 times

(f) $x' \leq 1 \lor x' \geq 5$ to be refined by $x := x + 1$

== $\forall x'. (x' \leq 1 \lor x' \geq 5) \iff x := x + 1$

== $\forall x'. (x' \leq 1 \lor x' \geq 5) \iff x' = x + 1$

== $x + 1 \leq 1 \lor x + 1 \geq 5$

== $\neg x: 1 \ldots 4$

== one-point

(h) $x' < y' \land \exists x. Lx < y'$ to be refined by $x := 1$

== $1 < y \land \exists x. Lx < y$

== $\exists y. Ly < x'$ to be refined by $x := y + 1$

== $\forall x', y', L'. (\exists y. Ly < x') \iff x' = y + 1 \land y' = y \land L' = L$

== $\exists x. Lx < y + 1$

== $L' 3 = 4$ to be refined by $L := i \rightarrow 4 \mid L$

== $\forall L', i'. (L' 3 = 4) \iff L := i \rightarrow 4 \mid L$

== $\forall L', i'. (L' 3 = 4) \iff L' = i \rightarrow 4 \mid L \land i' = i$

== $(i \rightarrow 4 \mid L) 3 = 4$

== $i = 3 \lor L 3 = 4$

== one-point twice

(j) $x' = a$ to be refined by if $a > b$ then $x := a$ else ok fi

== $\forall x', a', b'. x' = a \iff (a > b \land x' = a \land a' = a \land b' = b) \lor (a \leq b \land x' = x \land a' = a \land b' = b)$

== $\forall x', a', b'. (x' = a \iff a > b \land x' = x \land a' = a \land b' = b) \land (x' = a \iff a \leq b \land x' = x \land a' = a \land b' = b)$

== $(\forall x', a', b'. x' = a \iff a > b \land x' = a \land a' = a \land b' = b)$

== specialization and identity; one-point
\[ x = a \iff a \leq b \]

\[(l)\]
\[ x' = y \land y' = x \text{ to be refined by } (z := x. \ x := y. \ y := z) \]
\[ \forall x', y', z'. x' = y \land y' = x \iff (z := x. \ x := y. \ y := z) \]
\[ \forall x', y', z'. x' = y \land y' = x \iff (z := x. \ x := y. \ y := z) \land z' = z \]
\[ \text{Substitution Law} \]

\[(m)\]
\[ ax^2 + bx + c = 0 \text{ to be refined by } (x := ax + b. \ x := \neg x/a) \]
\[ \forall x'. \ ax^2 + bx + c = 0 \iff (x := ax + b. \ x := \neg x/a) \]
\[ \text{Definition of } ax^2 + bx + c = 0 \]

\[(n)\]
\[ f' = n! \text{ to be refined by } (n := n + 1. \ f := fn) \text{ where } n \text{ is natural and } ! \text{ is factorial.} \]
\[ \forall f', n'. f' = n! \iff (n := n + 1. \ f := fn) \]
\[ \forall f', n'. f' = n! \iff (n := n + 1. \ f := fn \land n' = n) \]
\[ \text{Substitution Law} \]

\[(o)\]
\[ 7 \leq c < 28 \land \text{odd } c \text{ to be refined by } (a := b - 1. \ b := a + 3. \ c := a + b) \]
\[ \forall a', b', c'. 7 \leq c < 28 \land \text{odd } c' \iff (a := b - 1. \ b := a + 3. \ c := a + b) \]
\[ \text{Definition of twice} \]

\[(p)\]
\[ s' = \Sigma L [0;..i'] \text{ to be refined by } (s := s + L i. \ i := i + 1) \]
\[ \forall s', i', L'. (s' = \Sigma L [0;..i']) \iff s' = s + Li \land i' = i + 1 \land L' = L \]

\[(q)\]
\[ x > 5 \text{ to be refined by } x': x + (1, 2) \]
\[ \forall x'. \ x > 5 \iff x' = x + (1, 2) \]
\[ \forall x'. \ x > 5 \iff x' = x + 1, x + 2 \]
\[ \text{Distributes over} \]

\[(r)\]
\[ x > 0 \text{ to be refined by } x': x + (-1, 1) \]
\[ \forall x'. \ x > 0 \iff x' = x + (-1, 1) \]
\[ \forall x'. \ x > 0 \iff x' = x - 1, x + 1 \]

\[ \text{Distributes over} \]
∀x' : x' > 0 ⇐ x' : x−1 ∨ x' : x+1

elementary axiom twice

∀x' : x' > 0 ⇐ x' = x−1 ∨ x' = x+1

antidistribution

∀x' : (x' > 0 ⇐ x' = x−1) ∧ (x' > 0 ⇐ x' = x+1)

distribution

(∀x' : x' > 0 ⇐ x' = x−1) ∧ (∀x' : x' > 0 ⇐ x' = x+1)

one-point twice

x−1 > 0 ∧ x+1 > 0

arithmetic

x>1 ∧ x≥−1

absorption

x>1