Let $x$ and $y$ be real variables. Prove that if $y = x^2$ is true before

$$x := x + 1. \quad y := y + 2x - 1$$

then it is still true after.

§ Here is one solution.

\[
\forall x', y'. \quad y' = x'^2 \iff (x := x + 1. \quad y := y + 2x - 1) \quad \text{expand final assignment}
\]

\[
\forall x', y'. \quad y' = x'^2 \iff (x := x + 1. \quad x' = x \land y' = y + 2x - 1) \quad \text{substitution law}
\]

\[
\forall x', y'. \quad y' = x'^2 \iff x' = x + 1 \land y' = y + 2x(x + 1) - 1 \quad \text{one-point law}
\]

\[
y + 2x(x + 1) - 1 = (x + 1)^2 \quad \text{arithmetic}
\]

\[
y = x^2
\]

Here is another solution.

\[
\forall x, y. \quad y = x^2 \iff (x := x + 1. \quad y := y + 2x - 1) \quad \text{expand final assignment}
\]

\[
\forall x, y. \quad y = x^2 \iff (x := x + 1. \quad x' = x \land y' = y + 2x - 1) \quad \text{substitution law}
\]

\[
\forall x, y. \quad y = x^2 \iff x' = x + 1 \land y' = y + 2x(x + 1) - 1 \quad \text{one-point law for } y
\]

\[
\forall x. \quad y' = 2x(x' - 1) + 1 = (x' - 1)^2 \quad \text{arithmetic}
\]

\[
y' = x'^2
\]

Here is yet another solution.

\[
y = x^2 \land (x := x + 1. \quad y := y + 2x - 1) \quad \text{expand final assignment}
\]

\[
y = x^2 \land (x := x + 1. \quad x' = x \land y' = y + 2x - 1) \quad \text{substitution law}
\]

\[
y = x^2 \land x' = x + 1 \land y' = y + 2x(x + 1) - 1 \quad \text{context}
\]

\[
y = x^2 \land x' = x + 1 \land y' = x^2 + 2x(x + 1) - 1 \quad \text{simplify}
\]

\[
y = x^2 \land x' = x + 1 \land y' = (x + 1)^2 \quad \text{context}
\]

\[
y = x^2 \land x' = x + 1 \land y' = x^2 \quad \text{specialization}
\]

\[
\Rightarrow y' = x'^2
\]

Here is even one more solution. This one works only because $(x := x + 1. \quad y := y + 2x - 1)$ is both implementable and deterministic.

\[
x := x + 1. \quad y := y + 2x - 1. \quad y = x^2 \quad \text{substitution law}
\]

\[
x := x + 1. \quad y + 2x - 1 = x^2 \quad \text{substitution law again}
\]

\[
y + 2x(x + 1) - 1 = (x + 1)^2 \quad \text{arithmetic}
\]

\[
y = x^2
\]