12 Formalize each of the following statements. For each of the ten pairs of statements, either prove they are equivalent or prove they differ.

(a) Don't drink and drive.
(b) If you drink, don't drive.
(c) If you drive, don't drink.
(d) Don't drink and don't drive.
(e) Don't drink or don't drive.

After trying the question, scroll down to the solution.
(a) Don't drink and drive.
\[ \neg (drink \land drive) \]

(b) If you drink, don't drive.
\[ drink \Rightarrow \neg drive \]
Proof that (a)= (b):
\[ \neg (drink \land drive) \]
\[ \equiv \neg drink \lor \neg drive \quad \text{duality} \]
\[ \equiv \neg \neg drive \Rightarrow \neg drive \quad \text{inclusion (material implication)} \]
\[ \equiv drive \Rightarrow \neg drive \quad \text{double negation} \]

(c) If you drive, don't drink.
\[ drive \Rightarrow \neg drink \]
Proof that (b)= (c):
\[ drink \Rightarrow \neg drive \quad \text{contrapositive} \]
\[ \equiv \neg drive \Rightarrow \neg drink \]
\[ \equiv drive \Rightarrow \neg drink \quad \text{double negation} \]

(d) Don't drink and don't drive.
\[ \neg drink \land \neg drive \]
Let \( drink \) be \( \top \) and let \( drive \) be \( \bot \). Then
\[ (a) \equiv \neg (drink \land drive) \equiv \neg (\top \land \bot) \equiv \bot \equiv \top \]
\[ (d) \equiv \neg drink \land \neg drive \equiv \neg \top \land \neg \bot \equiv \bot \land \top \equiv \bot \]

(e) Don't drink or don't drive.
\[ \neg drink \lor \neg drive \]
Proof that (a)= (e):
\[ \neg (drink \land drive) \]
\[ \equiv \neg drink \lor \neg drive \quad \text{duality} \]

We have proved (a)= (b) and (b)= (c) and (a)= (e), so (a), (b), (c), and (e) are all equal. We have proved (a) differs from (d), so (d) differs from (b), (c), and (e) as well.