12 Formalize each of the following statements. For each of the ten pairs of statements, either prove they are equivalent or prove they differ.

(a) Don't drink and drive.

(b) If you drink, don't drive.

(c) If you drive, don't drink.

(d) Don't drink and don't drive.

(e) Don't drink or don't drive.

After trying the question, scroll down to the solution.

(a) Don't drink and drive.

§ $\neg(drink \wedge drive)$

(b) If you drink, don't drive.

§ $drink \Rightarrow \neg drive$

Proof that (a)=(b):

	$\neg(drink \wedge drive)$	duality
$=$	$\neg drink \vee \neg drive$	inclusion (material implication)
$=$	$\neg\neg drink \Rightarrow \neg drive$	double negation
$=$	$drink \Rightarrow \neg drive$	

(c) If you drive, don't drink.

§ $drive \Rightarrow \neg drink$

Proof that (b)=(c):

	$drink \Rightarrow \neg drive$	contrapositive
$=$	$\neg\neg drive \Rightarrow \neg drink$	double negation
$=$	$drive \Rightarrow \neg drink$	

(d) Don't drink and don't drive.

§ $\neg drink \wedge \neg drive$

Let $drink$ be $\top$ and let $drive$ be $\bot$. Then

(a) $= \neg(drink \wedge drive) = \neg(\top \wedge \bot) = \neg\bot = \top$

(d) $= \neg drink \wedge \neg drive = \neg\top \wedge \neg\bot = \bot \wedge \top = \bot$

(e) Don't drink or don't drive.

§ $\neg drink \vee \neg drive$

Proof that (a)=(e):

	$\neg(drink \wedge drive)$	duality
$=$	$\neg drink \vee \neg drive$	

We have proved (a)=(b) and (b)=(c) and (a)=(e), so (a), (b), (c), and (e) are all equal. We have proved (a) differs from (d), so (d) differs from (b), (c), and (e) as well.