

119 (pigeon-hole) Let  $n$  be natural and let  $L$  be a finite list of finite numbers. Prove  
 $\sum L > n \times \#L \Rightarrow \exists i: \square L \cdot L i > n$

After trying the question, scroll down to the solution.

§    **if**  $\#L=0$  **then**     $\Sigma L > n \times \#L$     context

$=$      $0 > 0$

$=$      $\perp$

$\Rightarrow$   $\exists i: \square L \cdot L i > n$

**else**     $\Sigma L > n \times \#L$

$=$      $(\Sigma j \cdot L j) > n \times \#L$     increase  $L j$  to  $\uparrow L$

$\Rightarrow$   $(\Sigma j \cdot \uparrow L) > n \times \#L$

$=$      $\uparrow L \times \#L > n \times \#L$     cancellation

$=$      $\uparrow L > n$     generalization (for finite  $\#L$ )

$\Rightarrow$   $\exists i: L i > n$     **fi**