Relation $R$ is transitive if $\forall x, y, z: R x y \land R y z \Rightarrow R x z$. Express formally that relation $R$ is the transitive closure of relation $Q$ ($R$ is the strongest transitive relation that is implied by $Q$).

§ Here is a straightforward solution. Let $TR$ mean that $R$ is a transitive relation. Formally,

$$T = \langle R: (X \to X \to \text{bin}) \to \forall x, y, z: X \cdot R x y \land R y z \Rightarrow R x z \rangle$$

Let $A \geq B$ mean that relation $A$ is everywhere as strong as relation $B$. Formally,

$$A \geq B = \forall x, y: X \cdot A x y \Rightarrow B x y$$

Then we can say that $R$ is the transitive closure of $Q$ as follows:

$$TR \land Q \geq R \land \forall A: X \to X \to \text{bin}: TA \land Q \geq A \Rightarrow R \geq A$$

Here is a nicer solution, but only for the special case $X = 0..n$ for some extended natural $n$. Let $P i j k$ mean “there is a path in $Q$ from $j$ to $k$ via zero or more intermediate nodes all of which are less than $i$”. Formally,

$$P 0 = Q$$

$$\forall i, j, k: P (i+1) j k = P i j k \lor P i i \land P i i k$$

Then we can say that $R$ is the transitive closure of $Q$ as follows:

$$R = P n$$

This simple definition leads to a beautiful algorithm for transitive closure.