The following statements are made:

All unicorns are white.
All unicorns are black.
No unicorn is both white and black.

Are these statements consistent? What, if anything, can we conclude about unicorns?

Let unicorn be all unicorns. Let white and black be predicates on unicorns. Then

All unicorns are white:
(a) $\forall u:\text{unicorn} \cdot \text{white } u$
All unicorns are black:
(b) $\forall u:\text{unicorn} \cdot \text{black } u$
No unicorn is both white and black:
(c) $\neg \exists u:\text{unicorn} \cdot \text{white } u \land \text{black } u$

Suppose we take (a), (b), and (c) as axioms.

\[
\begin{align*}
\top & \quad \text{(a), (b), and (c) are axioms} \\
& \equiv (\forall u:\text{unicorn} \cdot \text{white } u) \land (\forall u:\text{unicorn} \cdot \text{black } u) \land (\neg \exists u:\text{unicorn} \cdot \text{white } u \land \text{black } u) \\
& \quad \text{Using a duality law (deMorgan) on (c), we can change it to a universal quantification:} \\
& \equiv (\forall u:\text{unicorn} \cdot \text{white } u) \land (\forall u:\text{unicorn} \cdot \text{black } u) \land (\forall u:\text{unicorn} \cdot \neg(\text{white } u \land \text{black } u)) \\
& \quad \text{Now we can use a splitting law to combine the three main conjuncts} \\
& \equiv \forall u:\text{unicorn} \cdot (\text{white } u \land \text{black } u) \land \neg(\text{white } u \land \text{black } u) \quad \text{Law of Noncontradiction} \\
& \equiv \forall u:\text{unicorn} \cdot \bot \quad \text{one-case} \\
& \equiv \text{if } \text{unicorn}=\text{null} \text{ then } \forall u:\text{unicorn} \cdot \bot \text{ else } \forall u:\text{unicorn} \cdot \bot \text{ fi} \\
& \quad \text{In the then-part, use if-part as context, and quantifier law } \forall v:\text{null} \cdot b . \\
& \quad \text{In the else-part, use negation of if-part as context, and idempotent law.} \\
& \equiv \text{if } \text{unicorn}=\text{null} \text{ then } \top \text{ else } \bot \text{ fi} \quad \text{there ought to be a law} \\
& \equiv \text{unicorn}=\text{null} \\
\end{align*}
\]

If we are given (a), (b), and (c) as axioms, we must conclude that there are no unicorns.