Consider a fully parenthesized expression containing only the symbols $T \bot = \pm ( )$ in any quantity and any syntactically acceptable order.

§ The proofs will be by induction over the structure of the expressions. Every fully parenthesized expression containing only the symbols $T \bot = \pm ( )$ has one of the following four forms: $T$, $\bot$, $(a=b)$, $(a+b)$, where $a$ and $b$ are fully parenthesized expression containing only the symbols $T \bot = \pm ( )$.

(a) Show that all syntactically acceptable rearrangements are equivalent.

There are four alternatives. The first two alternatives are just a single symbol, so there are no rearrangements, so all zero rearrangements are equivalent. That's the base case. Now for the induction step.

Suppose the expression is $(a=b)$ for some expressions $a$ and $b$. The ways of rearranging $(a=b)$ are:
(a) rearrange $a$
(b) rearrange $b$
(c) change $(a=b)$ to $(b=a)$
First, consider (a). Make the inductive hypothesis that rearranging $a$ results in an expression that is equivalent to $a$. Then any expression with subexpression $a$ is equivalent to the same expression with subexpression $a$ replaced by its rearrangement. (This is formalized as the generic law of transparency.) Similarly for (b). For (c), we have the generic law of symmetry of $=$.

That completes the proof for expressions of the form $(a=b)$. Finally, suppose the expression is $(a+b)$ for some expressions $a$ and $b$. The ways of rearranging $(a+b)$ are:
(a) rearrange $a$
(b) rearrange $b$
(c) change $(a+b)$ to $(b+a)$
First, consider (a). Make the inductive hypothesis that rearranging $a$ results in an expression that is equivalent to $a$. Then any expression with subexpression $a$ is equivalent to the same expression with subexpression $a$ replaced by its rearrangement. (This is formalized as the generic law of transparency.) Similarly for (b). For (c), we have the generic law of symmetry of $\pm$.

That completes the proof.

(b) Show that it is equivalent to any expression obtained from it by making an even number of the following substitutions: $T$ for $\bot$, $\bot$ for $T$, $=$ for $\neq$, $\neq$ for $=.$

§ Zero substitutions means the same expression, which is obviously equivalent. I will show that by making a single one of those substitutions, the expression is negated. Therefore two substitutions are a double negation, which is an equivalent expression. And so on for more substitutions.

If the expression is $T$, the only substitution is $\bot$ for $T$, and $\bot$ is the negation of $T$.

If the expression is $\bot$, the only substitution is $T$ for $\bot$, and $T$ is the negation of $\bot$.
Suppose the expression is \((a=b)\) for some expressions \(a\) and \(b\). The ways of making one substitution in \((a=b)\) are:

(a) make one substitution in \(a\)
(b) make one substitution in \(b\)
(c) change \((a=b)\) to \((a\neq b)\)

First, consider (a). Make the inductive hypothesis that one substitutions in \(a\) negates \(a\), resulting in an expression equivalent to \((\neg a=b)\).

\[
\begin{align*}
(\neg a=b) & \quad \text{exclusion} \\
= & \quad (a\neq b) \\
= & \quad \neg(a=b)
\end{align*}
\]

so making one substitution in \(a\) negates \((a=b)\). Similarly for (b). For (c),

\[
\begin{align*}
(a\neq b) & \quad \text{generic inequality} \\
= & \quad \neg(a=b)
\end{align*}
\]

That completes the proof for expressions of the form \((a=b)\). Finally, suppose the expression is \((a\neq b)\) for some expressions \(a\) and \(b\). The ways of making one substitution in \((a\neq b)\) are:

(a) make one substitution in \(a\)
(b) make one substitution in \(b\)
(c) change \((a\neq b)\) to \((a=b)\)

First, consider (a). Make the inductive hypothesis that one substitutions in \(a\) negates \(a\), resulting in an expression equivalent to \((\neg a\neq b)\).

\[
\begin{align*}
(\neg a\neq b) & \quad \text{generic inequality} \\
= & \quad \neg(\neg a=b) \\
= & \quad \neg(a\neq b)
\end{align*}
\]

so making one substitution in \(a\) negates \((a\neq b)\). Similarly for (b). For (c),

\[
\begin{align*}
(a=b) & \quad \text{double negation} \\
= & \quad \neg(\neg a=b) \\
= & \quad \neg(a\neq b)
\end{align*}
\]

That completes the proof.