

Here is an informal explanation of the one-point laws. Let's start with

$$\exists v: D \cdot v=x \wedge b$$

and we are given $x: D$. Let's suppose D is *nat*. The existential quantification is an infinite disjunction.

$$(0=x \wedge b) \vee (1=x \wedge b) \vee (2=x \wedge b) \vee \dots$$

We can use context in each of these disjuncts.

$$\begin{aligned} &(0=x \wedge (b \text{ but replace } x \text{ with } 0)) \\ &\vee (1=x \wedge (b \text{ but replace } x \text{ with } 1)) \\ &\vee (2=x \wedge (b \text{ but replace } x \text{ with } 2)) \\ &\vee \dots \end{aligned}$$

Since x is in *nat*, exactly one of $0=x, 1=x, 2=x, \dots$ is \top and the others are \perp . That's why it's called "one-point". Let's suppose x is 1.

$$\begin{aligned} &(\perp \wedge (b \text{ but replace } x \text{ with } 0)) \\ &\vee (\top \wedge (b \text{ but replace } x \text{ with } 0)) \\ &\vee (\perp \wedge (b \text{ but replace } x \text{ with } 0)) \\ &\vee \dots \\ = &\perp \vee (b \text{ but replace } x \text{ with } 1) \vee \perp \vee \dots \\ = &(b \text{ but replace } x \text{ with } 1) \end{aligned}$$

Now the other one.

$$\forall v: D \cdot v=x \Rightarrow b$$

and we are given $x: D$. Let's suppose D is *nat*. The universal quantification is an infinite conjunction.

$$(0=x \Rightarrow b) \wedge (1=x \Rightarrow b) \wedge (2=x \Rightarrow b) \wedge \dots$$

We can use context in each of these conjuncts.

$$\begin{aligned} &(0=x \Rightarrow (b \text{ but replace } x \text{ with } 0)) \\ &\wedge (1=x \Rightarrow (b \text{ but replace } x \text{ with } 1)) \\ &\wedge (2=x \Rightarrow (b \text{ but replace } x \text{ with } 2)) \\ &\wedge \dots \end{aligned}$$

Since x is in *nat*, exactly one of $0=x, 1=x, 2=x, \dots$ is \top and the others are \perp . That's why it's called "one-point". Let's suppose x is 1.

$$\begin{aligned} &(\perp \Rightarrow (b \text{ but replace } x \text{ with } 0)) \\ &\wedge (\top \Rightarrow (b \text{ but replace } x \text{ with } 0)) \\ &\wedge (\perp \Rightarrow (b \text{ but replace } x \text{ with } 0)) \\ &\wedge \dots \\ = &\top \wedge (b \text{ but replace } x \text{ with } 1) \wedge \top \wedge \dots \\ = &(b \text{ but replace } x \text{ with } 1) \end{aligned}$$