BUNCH THEORY

Bunches can be used to represent collections.

Examples: 1, 3, 7 2

Any number, character, boolean, or set is an <u>elementary bunch</u>, or <u>element</u>.

If A and B are bunches, then

A , B	union
A ‘ B	intersection

are bunches,

¢A	size, cardinality

is a number, and

$A: B$	inclusion

is a boolean.

1, 3, 7 = 3, 1, 7, 1

¢2 = 1

¢(0, 2, 5, 9) = 4

2: 0, 2, 5, 9

2: 2

2, 9: 0, 2, 5, 9

axioms

$x\colon y \;=\; x=y$	elementary axiom
$x\colon A,B \;=\; x\colon A \;\vee\; x\colon B$	compound axiom
$A,A = A$	idempotence
$A,B = B,A$	symmetry
$A,(B,C) = (A,B),C$	associativity
$A\text{‘}A = A$	idempotence
$A\text{‘}B = B\text{‘}A$	symmetry
$A\text{‘}(B\text{‘}C) = (A\text{‘}B)\text{‘}C$	associativity
$A,B\colon C \;=\; A\colon C \;\wedge\; B\colon C$	
$A\colon B\text{‘}C \;=\; A\colon B \;\wedge\; A\colon C$	
$A\colon A,B$	generalization
$A\text{‘}B\colon A$	specialization
$A\colon A$	reflexivity
$A\colon B \;\wedge\; B\colon A \;=\; A=B$	antisymmetry
$A\colon B \;\wedge\; B\colon C \;\Rightarrow\; A\colon C$	transitivity
$\text{¢}x = 1$	size
$\text{¢}(A,B) + \text{¢}(A\text{‘}B) = \text{¢}A + \text{¢}B$	size
$\neg\, x\colon A \;\Rightarrow\; \text{¢}(A\text{‘}x) = 0$	
$A\colon B \;\Rightarrow\; \text{¢}A \le \text{¢}B$	

laws

$A,(A‘B) = A$	absorption
$A‘(A,B) = A$	absorption
$A: B \Rightarrow C,A: C,B$	monotonicity
$A: B \Rightarrow C‘A: C‘B$	monotonicity
$A: B = A,B = B = A = A‘B$	inclusion
$A,(B,C) = (A,B),(A,C)$	distributivity
$A,(B‘C) = (A,B)‘(A,C)$	distributivity
$A‘(B,C) = (A‘B), (A‘C)$	distributivity
$A‘(B‘C) = (A‘B)‘(A‘C)$	distributivity
$A: B \wedge C: D \Rightarrow A,C: B,D$	conflation
$A: B \wedge C: D \Rightarrow A‘C: B‘D$	conflation

null			the empty bunch
bool	=	$\top, \bot$	the booleans
nat	=	0, 1, 2, ...	the natural numbers
int	=	..., –2, –1, 0, 1, 2, ...	the integer numbers
rat	=	..., –1, 0, 2/3, ...	the rational numbers
real	=	...	the real numbers
xnat	=	0, 1, 2, ..., ∞	the extended naturals
xint	=	$-\infty$, ..., –2, –1, 0, 1, 2, ..., ∞	the extended integers
xrat	=	$-\infty$, ..., –1, 0, 2/3, ..., ∞	the extended rationals
xreal	=	$-\infty$, ..., ∞	the extended reals
char	=	..., \`a, \`A, ...	the characters

$x,..y$ “ x to y ” for $x \le y$

$$0,..5 \;=\; 0, 1, 2, 3, 4$$

$$0,..\infty \;=\; nat$$

$$5,..5 \;=\; null$$

$$¢(x,..y) \;=\; y-x$$

Distribution

$-(1, 3, 7) \;=\; -1, -3, -7$

$(1, 2) + (10, 20) \;=\; 11, 12, 21, 22$

$(1, 2) + 10 \;=\; 11, 12$

$1 + 10 \;=\; 11$

$\mathit{null} + 10 \;=\; \mathit{null}$

$\mathit{nat} + 2 \;=\; 2, 3, 4, 5, 6, ...$

$\mathit{nat} \times 2 \;=\; 0, 2, 4, 6, 8, ...$

$\mathit{nat}^2 \;=\; 0, 1, 4, 9, 16, ...$

$2^{\mathit{nat}} \;=\; 1, 2, 4, 8, 16, ...$

SET THEORY

{*A*}	“set containing *A* ”
~*S*	“contents of *S* ”

{1, 3, 7}

{1, {3, 7}}

{*null*}	the empty set
{*nat*}	the set of natural numbers

{0, 1, 2} = {0,..3}

~{1, 3, 7} = 1, 3, 7

${1, 3, 7} = 3

2{0, 1} = {{*null*}, {0}, {1}, {0, 1}} powerset

axioms

{*A*} ǂ *A*

~{*A*} = *A*

$\{*A*} = ¢*A*

A ∈ {*B*} = *A*: *B*

{*A*} ⊆ {*B*} = *A*: *B*

{*A*} ∈ 𝟚{*B*} = *A*: *B*

{*A*} ∪ {*B*} = {*A*, *B*}

{*A*} ∩ {*B*} = {*A* ‘ *B*}

{*A*} = {*B*} = *A* = *B*

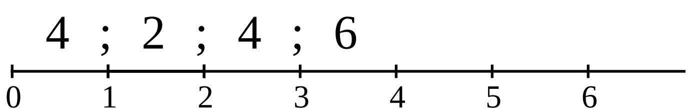
STRING THEORY

nil — the empty string

2

4; 2; 4; 6

$\#(4;\ 2;\ 4;\ 6)\ =\ 4$



$(3;\ 5;\ 7;\ 9)_2\ =\ 7$

$(3;\ 5;\ 7;\ 9)_{2;\ 1;\ 2}\ =\ 7;\ 5;\ 7$

$3;\ 6;\ 4;\ 7\ <\ 3;\ 7;\ 2$

$3;\ 6;\ 4\ <\ 3;\ 6;\ 4;\ 7$

$x;..y$ — " x to y " for $x \le y$

$\#(x;..y)\ =\ y-x$

$(x;..y)\ ;\ (y;..z)\ =\ x;..z$

0; 1; 2: *nat*; 1; (0,..10)

$3*(4;\ 5)\ =\ 4;\ 5;\ 4;\ 5;\ 4;\ 5$

$*3\ =\ $ *nil* , 3 , 3;3 , 3;3;3 , ...

LIST THEORY

[0; 1; 2]

[0; 1; 2]:   [*nat*; 1; (0,..10)]:   [3**nat*]:   [**nat*]

~[3; 5; 7; 4] = 3; 5; 7; 4

#[3; 5; 7; 4] = 4

[3; 5; 7; 4] 2 = 7

[3; 5; 7; 4] [2; 1; 2] = [7; 5; 7]

[3; 5; 7; 4]+[2; 1; 2] = [3; 5; 7; 4; 2; 1; 2]

2→22 | [10;..15] = [10; 11; 22; 13; 14]

Let *L* = [10;..15] . Then

2→*L*3 | 3→*L*2 | *L*   =   [10; 11; 13; 12; 14]

[3; 6; 4; 7] < [3; 7; 2]

[3; 6; 4] < [3; 6; 4; 7]

"Don't say ""no""."

=   [\`D; \`o; \`n; \`'; \`t; \` ; \`s; \`a; \`y; \` ; \`"; \`n; \`o; \`"; \`.]

"abcdefghij" [3;..6] = "def"

$L\ (n, m)\ =\ Ln, Lm$

$L\ \{n, m\}\ =\ \{Ln, Lm\}$

$L\ (n; m)\ =\ Ln; Lm$

$L\ [n; m]\ =\ [Ln; Lm]$

$L\ [0, \{1, [2; 1]; 0\}]\ =\ [L0, \{L1, [L2; L1]; L0\}]$

Multidimensional Structures

A = [[6; 3; 7; 0] ;
[4; 9; 2; 5] ;
[1; 5; 8; 3]]

A: [3*[4*nat]]

A 1 = [4; 9; 2; 5]

A 1 2 = 2

A (1, 2) = A 1, A 2 = [4; 9; 2; 5], [1; 5; 8; 3]

A [1, 2] = [A 1, A 2] = [[4; 9; 2; 5], [1; 5; 8; 3]]

$$B \;=\; [\ [2;\ 3];\ 4;\ [5;\ [6;\ 7]\ ]\ ]$$

$$B\ 0\ 0 = 2$$

$$B\ 1 = 4$$

$$B\ 1\ 1$$

$$L@nil \;=\; L$$

$$L@n \;=\; L\ n$$

$$L@(S;\ T) \;=\; L@S@T$$

$$B@(2;\ 1;\ 0) \;=\; B\ 2\ 1\ 0 \;=\; 6$$

$$nil \rightarrow i \mid L \;=\; i$$

$$(S;T) \rightarrow i \mid L \;=\; S \rightarrow (T \rightarrow i \mid L@S) \mid L$$

$$(0;1) \rightarrow 6 \mid [\ [0;\ 1;\ 2]\ ;\ [3;\ 4;\ 5]\ ] \;=\; [\ [0;\ 6;\ 2]\ ;\ [3;\ 4;\ 5]\ ]$$