a Conversation between a Platonist and a Formalist

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Platonist: Mathematical objects, like numbers and sets and functions, exist. You can't see them or feel them or sense them in any way, but they exist. They are abstract objects. There are facts or truths about these objects, and a mathematician's job is to discover these truths. But the truths are independent of the existence of people; they were true before there were any people, and they will still be true after there are no more people.

Formalist: Mathematics is a language whose expressions can be used to describe or model some aspects of the world, especially quantitative aspects. By themselves the expressions are neither true nor false, but each application of mathematics supplies facts or truths that are represented by mathematical expressions. The design and use of the language of mathematical expressions constitute the subject. Mathematical language can include diagrams, graphs, and anything else; usually and most usefully the expressions are sequences of symbols, providing a means for calculation and reasoning. Mathematics is a human creation.

Platonist: $1+1=2$ is a fact, independent of people. It would be true even if there were no people.

Formalist: 2 is defined as $1+1$. And then 3 is defined as $2+1$. And 4 is defined as $3+1$. And so on. So $1+1=2$ is true by definition, and definitions are human creations, not facts independent of people.

Platonist: Humans have defined the word “two” and the symbol 2, but not the mathematical object that it stands for.

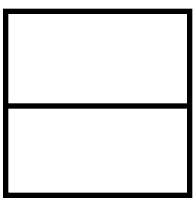
Formalist: What object? Show it to me.

Platonist: Ok, let's look at a more interesting fact. It is a fact that the ratio of the circumference of a circle to its diameter is π. People did not decide to make that ratio be π. That would be a fact even if there were no people.

Formalist: The definition of π is: the ratio of the circumference of a circle to its diameter. And that definition is a human creation.

Platonist: I meant that the ratio of the circumference of a circle to its diameter is approximately 3.14. That is a fact independent of people, not a human creation. And, amazingly, it's that same ratio no matter how big or little the circle is. If the diameter is 1 cm, the ratio of circumference to diameter is approximately 3.14. If the diameter is 1 km, the ratio is still approximately 3.14.

Formalist: Let me look at a similar example. Here is a square, and its diameter.



The ratio of the circumference to the diameter is 4. Part of the definition of “square” is that it has 4 equal sides, “circumference” is defined as the sum of the lengths of the sides, and

“diameter” seems to be defined as being the length of one side. So it is a consequence of the definitions of “square” and “circumference” and “diameter” that the ratio of circumference to diameter is 4. Definitions are made by people. The picture does not show the scale. Each side, and the diameter, could be 1 cm, or they could be 1 km. There are still 4 sides, so the ratio is still 4. That doesn't seem amazing to me. A circle with its diameter is a little more complicated, but it is still a consequence of the definitions of “circle” and “diameter” that the ratio is approximately 3.14.

Platonist: That picture, or any picture you might draw of a square or circle, is not a mathematical square or circle. It is just a human's approximation. A mathematical square or circle is a line with zero width, so it's invisible, and it is perfectly placed with no tiny deviations due to the finite pixel density. A mathematical object is an ideal abstraction.

Formalist: Nice words. But you have to say what you mean by a perfectly drawn zero width line. And you can do that with a formula. For a circle, the formula is $x^2 + y^2 = r^2$. A formula is something we can see. It's not an abstract object, whatever that is. It's the formula, which is a human creation, together with the rules of calculation, which are a human creation, that enable us to calculate $\boldsymbol{\pi} = 3.14$ (approximately).

Platonist: When we sent the Pioneer spacecraft off to other parts of our galaxy, we included some mathematics because math is universal. Any intelligent life on other planets won't have the same languages we have, but they will have the same math we have. The highly honored mathematician Sir Roger Penrose has said exactly that.

Formalist: That is an amazingly egotistical and ridiculous assumption. Anyway, we don't need to go to other planets to find other forms of intelligent life; there's plenty here on Earth. When a scout bee comes back to the hive, it tells the other bees the direction and distance to the flowers it found. To us, it looks like a dance, but there may be more to the communication than we know. Somehow, bees communicate quantitative information, so that's bee math. We don't understand bee math, and bees don't understand our math. So why do you suppose life on another planet would have the same math as us?

Platonist: I say math is universal, and the vast majority of mathematicians agree with me.

Formalist: Yes, they do. But they are wrong.

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