a Conversation between a Platonist and a Formalist

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2022-7-18

Platonist: Mathematical objects, like numbers and sets and functions, exist. You can't see them or feel them or sense them in any way, but they exist. They are abstract objects. There are facts or truths about these objects, and a mathematician's job is to discover these truths. The truths are independent of the existence of people; they were true before there were any people, and they will still be true after there are no more people.

Formalist: Mathematics is a human creation. It is a language whose expressions can be used to describe or model some aspects of the world, especially quantitative aspects. By themselves the expressions are neither true nor false, but each application of mathematics supplies facts or truths that are represented by mathematical expressions. The design and use of the language of mathematical expressions constitute the subject. Mathematical language can include diagrams, graphs, and anything else; usually and most usefully the expressions are sequences of symbols, providing a means for calculation and reasoning.

Platonist: $1+1=2$ is a fact, independent of people. It would be true even if there were no people.

Formalist: 2 is defined as $1+1$. And then 3 is defined as $2+1$. And 4 is defined as $3+1$. And so on. So $1+1=2$ is true by definition, and definitions are human creations, not facts independent of people.

Platonist: Humans have defined the word “two” and the symbol “2”, but not the mathematical object that it stands for.

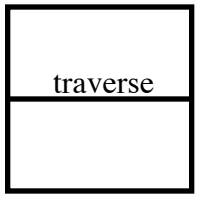
Formalist: What object? Show it to me. You can show me the mathematical symbol 2. You can show me 2 apples, applying the symbol 2 to something in the real, physical world. But you cannot show me the mathematical object 2.

Platonist: I can show you 2 apples, and 2 oranges, and 2 elephants. What these exhibits have in common is the abstract number 2. But let's look at a more interesting example. It is a fact that the ratio of the circumference of a circle to its diameter is π. People did not decide to make that ratio be π. That would be a fact even if there were no people.

Formalist: The definition of π is: the ratio of the circumference of a circle to its diameter. And that definition is a human creation.

Platonist: I meant that the ratio of the circumference of a circle to its diameter is approximately 3.14. That is a fact independent of people, not a human creation. And, amazingly, it's that same ratio no matter how big or little the circle is. If the diameter is 1 cm, the ratio of circumference to diameter is approximately 3.14. If the diameter is 1 km, the ratio is still approximately 3.14.

Formalist: Let me look at a similar example. Here is a square. Define a traverse of a square to be a line from one side to the opposite side, parallel to the remaining sides, as in the picture.



The ratio of the circumference to the traverse is 4. Part of the definition of "square" is that it has 4 equal sides, "circumference" is defined as the sum of the lengths of the sides, and "traverse" is the length of one side. So it is a consequence of the definitions of "square" and "circumference" and "traverse" that the ratio of circumference to traverse of a square is 4. Definitions are made by people. The picture does not show the scale. Each side, and the traverse, could be 1 cm, or they could be 1 km. There are still 4 sides, so the ratio is still 4. That doesn't seem amazing to me. A circle with its diameter is a little more complicated, but it is still a consequence of the definitions of "circle", "circumference", and "diameter" that the ratio is approximately 3.14.

Platonist: That picture, or any picture you might draw of a square or circle, is not a mathematical square or circle. It is just a human's approximation. A mathematical square or circle is a line with zero width, so it's invisible, and there are no tiny deviations due to the finite pixel density. A mathematical object is an ideal abstraction.

Formalist: Nice words. But you have to say what you mean by a perfect zero-width line. And you can do that with an expression. For a circle, the expression is $x^2 + y^2 = r^2$. An expression is something we can see. It's not an abstract object, whatever that is. It's the expression, which is a human creation, together with rules of calculation, which are a human creation, that enable us to calculate $\pi = 3.14$ (approximately).

Platonist: You keep talking about expressions, but that's just the syntax of mathematics. You keep ignoring the semantics, the meaning, of the expressions, which is the objects the expressions refer to. Without the objects, the expressions don't mean anything, and mathematics is just arbitrary nonsense.

Formalist: In applied math, the expressions refer to real quantities in the application domain. The expressions are arbitrary in the same way that the words and grammar of a natural language are arbitrary. Mathematical expressions could have been designed differently, just as there are different natural languages. The expressions of applied math are not nonsense; the application domain supplies the sense, just as it does for natural language. However, so-called "pure math" is arbitrary nonsense. Or, to be kinder, it is a game without a purpose. Applied math could be renamed "useful math", and pure math could be renamed "useless math".

Platonist: In my view, mathematics is nobler than your low-level "engineering" view. I prefer to think that my work is timeless and universal.

Formalist: Indeed, I see an analogy between your view of mathematics and religion. A religious person believes that God and angels and souls exist, even though you can't see them or sense them in any way, just as you believe in mathematical objects. A religious person believes these objects give meaning to life, and make their lives nobler. They feel connected to something timeless and universal. You are a like a priest or prophet; you believe you are the discoverer and keeper of the sacred mathematical truths. I am like an atheist; solving real problems here and now is noble enough for me. Like any analogy, it's not perfect, but there's some similarity.

Platonist: When we sent the Pioneer spacecraft off to other parts of our galaxy, we included some mathematics because math is universal. Any intelligent life on other planets won't have the same languages we have, but they will have the same math we have. The highly honored

mathematician Sir Roger Penrose has said exactly that.

Formalist: That is an amazingly egotistical and ridiculous assumption. Anyway, we don't need to go to other planets to find other forms of intelligent life; there's plenty here on Earth. When a scout bee comes back to the hive, she tells the other bees the direction and distance to the flowers she found. To us, it looks like a dance, but there may be more to the communication than we know. Somehow, bees communicate quantitative information, so that's bee math. We don't understand bee math, and bees don't understand our math. So why do you suppose life on another planet would have the same math as us?

Platonist: The eminent physicist Eugene Wigner wrote about "the unreasonable effectiveness of mathematics in the natural sciences" in 1960. His point was that the laws of nature, the mathematics that describes nature, apply not only to Earth, but to the entire universe. That is proof that mathematics is universal.

Formalist: In that article, Wigner wrote "the laws of nature are written in the language of mathematics"; he is saying, as Galileo did before him, that mathematics is a language, not a collection of abstract objects. That language was invented by humans for the purpose of describing the world in a quantitative way, just as natural languages were invented by humans for describing the world in other ways. The reason for calling math "unreasonably effective" is the mistaken belief that math was not designed by us for that purpose, but just happens to be good for that purpose. Perhaps it's amazing that we have succeeded as well as we have.

Platonist: I say math is universal, and the vast majority of mathematicians agree with me.

Formalist: Yes, they do. But they are wrong.

Platonist: Now who is egotistical?

Formalist: I prefer to say self-confident.

Platonist: You say I'm egotistical and you're self-confident. I say it's the other way round.

[The conversation has now degenerated too much to continue.]

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