ProTem is a programming system that serves as both programming language and operating system, and includes a theorem prover to check each step of program composition. This document is an informal specification of ProTem. Formal specifications of the data types and program semantics can be found in the book *a Practical Theory of Programming* (with syntactic differences). “ProTem” also means “for now”, short for the Latin words “pro tempore”.

Programming languages and operating system languages have a lot of functionality in common, but differ greatly in syntax and terminology. These differences are historical, accidental, and unnecessary. They complicate a programmer's life with no benefit. For example, a file is just a variable; file update and storage are just assignment. By unifying the programming language and the operating system commands, both gain in functionality. Communication channels and file piping are as useful in programming as they are in operating systems. Directories and permissions are useful in large-scale multi-programmer programs. Conditional execution (if) and indexed loops (for) are useful operating system commands.

ProTem is also designed for easy proof of correctness, including functionality, time requirements, and space requirements. To that end, loops can be constructed by labeling any block of code with a specification, and then using the label within the block of code. For example,

\[
\langle n \geq 0 \Rightarrow n' = 0 \rangle \quad \text{if} \quad n > 0 \quad \langle n := n - 1. \quad \langle n \geq 0 \Rightarrow n' = 0 \rangle \rangle
\]

The proof methods are the subject of the book *a Practical Theory of Programming* and paper *Specified Blocks*. They do not require preconditions, postconditions, or invariants. If proof is not wanted, then an ordinary identifier can be used as label. For example,

\[
\text{loop} \quad \text{if} \quad n > 0 \quad \langle n := n - 1. \quad \text{loop} \rangle
\]

A primary design criterion is to make ProTem a small, easy-to-learn, easy-to-use language. The size of a language can be measured by the number of symbols and by the complexity of grammar structure, which can be measured by the number of nonterminals. ProTem has 8 keywords. (C has 28, Python has 35, Pascal has 36, Haskell has 37, Ada has 62, MS Basic has 205.) ProTem is presented by a Presentation Grammar, which has just the structure that a programmer needs to know, not all the structure that a compiler needs. It has 2 nonterminals (program and data) plus some informally defined kinds of names. (There is also an LL(1) grammar with 23 nonterminals and an LR(0) grammar with 13 nonterminals at the end of this document. For comparison, the Haskell grammar has 68 nonterminals, and the Python grammar has 87 nonterminals.) The design ethos demands an extremely good reason for adding a new feature to ProTem that requires a new keyword or syntax. That same design ethos will not tolerate any addition to the 2 nonterminals in the Presentation Grammar.

To judge ease of use, you need to use the language, but you may get a sense of the ease of use (and of the beauty of the language, if that’s of interest) from reading example programs. For that purpose, there are example programs near the end of this document.

The design of ProTem is complete except for the following. We need to describe and compose picture and sound elements. We need to define touchpad and touchscreen gestures. We may need to define regions of documents and regions of the screen to be clickable links. An implementation is partly written.
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Acknowledgements
Symbols

ProTem has 8 keywords, plus 5 kinds of lexeme, and 68 other symbols; altogether they are:

```
case  else  for  if  new  old  plan  result
```

```
number  text  name  comment  identity
```

```
"  "  «  »  `  '  ,  ,..  ;  ;;  ;..  .  :  ::  :=  =
```

```
≠  <  >  ≤  ≥
```

```
!  !!  ?  ??  (  )  { }  \[  \]
```

```
〈  〉  ⟦  ⟧
```

```
_  |  ||
```

```
∞
```

```
&  %  +  –  ×  /  \  →  ↔  ⊤  ⊥
```

```
∧  ∨  ^  ^^  @  *  ~
```

```
¢  #  #1  ☐  ⊲  ⊳  ⊨
```

```
⫤
```

Some of the ProTem symbols may not be on your keyboard. Here are the substitutes.

- for “ and ” use 
- for ‘ use 
- for ≠ use /=
- for ≤ use <=
- for ≥ use >=
- for 〈 use <: for 〉 use :>
- for \ use |
- for – use -
- for × use ><
- for ⌈ use / \ for ⌉ use / /
- for = use =!

The names `infinity`, `true`, and `false` are predefined, and redefinable.

A number is formed as one or more decimal digits, with an optional decimal point between digits. A decimal point must have at least one digit on each side of it. Here are four examples.

```
0     275     27.5     0.21
```

A text begins with a left-double-quote, continues with any number of any characters (but a double-quote (left or right) within a text must be underlined), and concludes with a right-double-quote. Characters within a text are not limited to any alphabet. Here are five examples.

```
""          "abc"         "don't"         "Just say "no"."         "♠♣♥♦"
```

A name is either simple or compound. A simple name is either plain or fancy. A plain simple name begins with a letter (from an alphabet), and continues with any number of letters and digits, except that keywords cannot be names. A fancy simple name begins with « , and continues with any number of any characters (not limited to any alphabet) except « and », and ends with ». Within a fancy simple name, blank spaces, tabs, and new lines are not significant. A compound name is composed of two or more simple names joined with underscore characters. For examples:

- plain simple names: x Al george refStack
- fancy simple names: «William & Mary» «x’ ≥ x»
- compound names: ProTem_grammars_LL1 DCS_«grad recruiting» «2016-9-8»

A comment begins with ` and ends at the end of the line. Characters within a comment are not limited to any alphabet. For example: ` I❤ProTem

An identity identifies a person. Ideally it is biometric, such as a fingerprint or iris scan. Or it could be an encoded (encrypted) password. It is used to grant access to programs and data.

Presentation Grammar

At each point in a program, a name is one of

- newname: a simple name that is not defined in the current scope,
- or a compound name that is not defined in its dictionary
- oldname: a simple name that is defined in the current scope,
- or a compound name that is defined in its dictionary
An oldname is defined as one of: variablename, constantname, dataname, programname, channelname, unitname, or dictionaryname.

There are 32 ways of forming a program. Each way will be explained later. Some examples and pronunciations are shown on the right side.

new newname : data := data  
new newname := data  
new newname = data  
new newname [ program ]  
new newname ? data ! data  
new newname #1  
new newname _  
new newname oldname  
new newname  
old oldname  
variablename := data  
channelname ! data  
channelname ? data  
channelname ? data ! channelname  
channelname ? ! channelname  
simplename [ program ]  
programname  
plan simplename : data [ program ]  
plan simplename :: data [ program ]  
plan simplename ! data [ program ]  
plan simplename ? data [ program ]  
program data  
program variablename  
program channelname  
program . program  
program || program  
if data [ program ]  
if data [ program ] else [ program ]  
case data [ program ]  
case data [ program ] else [ program ]  
for simplename := data [ program ]  
[ program ]

There are 56 ways of expressing data. Each way will be explained later. Some examples and pronunciations are shown on the right side.

number  
∞  
data & data  
data %  
+ data  
– data  
data + data  
data – data  
0 1.2  
∞  
infinity, the infinite number  
complex number, data +i× data  
percentage, divide by 100  
plus, identity  
minus, negation, not  
plus, addition  
minus, subtraction
data \times data  
\text{times, multiplication}  
data / data  
\text{divided by, division}  
data ^ \data  
\text{to the power, exponentiation}  
data ^{\wedge} data  
\text{scientific notation, data \times 10^ data}  
\top  
\text{top, true}  
\bot  
\text{bottom, false}  
data \land data  
\text{minimum, conjunction, and, set intersection}  
data \lor data  
\text{maximum, disjunction, or, set union}  
data = data  
\text{equals, equation}  
data \neq data  
\text{not equals, differs from, exclusive or}  
data < data  
\text{less than, strict implication, strict subset}  
data > data  
\text{greater than, strict reverse implication, strict superset}  
data \leq data  
\text{less than or equal to, implication, subset}  
data \geq data  
\text{greater than or equal to, reverse implication, superset}  
data , data  
\text{bunch union}  
data ... data  
\text{bunch from (including) to (excluding)}  
data \cdot data  
\text{bunch intersection}  
data : data  
\text{bunch inclusion}  
\varepsilon data  
\text{bunch size, cardinality}  
\{ data \}  
\text{contents of a set or list}  
\sim data  
\text{power}  
\textbf{“abc”}  
data \# data  
\text{string join}  
data \ldots data  
\text{string from (including) to (excluding)}  
data \backslash data  
\text{string indexing}  
data \leftrightarrow data  
\text{string modification}  
data \star data  
\text{string length}  
data * data  
\text{definite repetition}  
data * data  
\text{indefinite repetition}  
[ data ]  
\text{list}  
data ;; data  
\text{list join}  
data # data  
\text{list length, function size}  
data data  
\text{list index, function argument, composition}  
data @ data  
\text{pointer indexing}  
\langle \text{ simplename } : data \rightarrow data \rangle  
\text{function, parameter is constantname}  
data \rightarrow data  
\text{function, function space}  
\square data  
\text{domain of a list or function}  
data \downarrow data  
\text{selective union}  
\textbf{\textit{variblename}}  
\text{variable name}  
\textbf{\textit{constantname}}  
\text{constant name}  
\textbf{\textit{dataname}}  
\text{data name and evaluate data}  
\textbf{\textit{channelname}} ??  
\text{the most recent data read on the channel}  
\textbf{\textit{channelname}} !!  
\text{test for written but unread data on the channel}  
\textbf{\textit{unitname}}  
\text{unit name, positive finite real number constant}  
data \doteq data \doteq data  
\text{conditional data, if data then data else data}  
\textbf{result}  
\text{expression, create variblename}  
\textbf{result}\text{-expression, create variablename}  
data ( data )  
\text{data parentheses}
Here is the precedence (order of execution) of the forms of program.
0  new  old  :=  !  ?  programname  plan  if  case  for  [ ] ]
1  program  data  program  variablename  program  channelname  left-to-right
2  ||
3  .
Program parentheses [ ] can always be used to group programs differently.

Here is the precedence (order of evaluation) of the forms of data.
0  number  text  name  T  ⊥  ∞  ??  !!  ( )  [ ]  { }  ＜  ＞  result
1  data  data  post  fi  x  %  ??  !!  in  fi  x  @  &  left-to-right
2  pre  fi  x  +  –  ¢  ↔  #  ~  ☐  *  in  fi  x  *  \  →  ^  ^^  right-to-left
3  n  fi  x  ×  /  ∧  ∨  left-to-right
4  n  fi  x  +  –  ;;  ;  ;..  |  ⊲ ⊳  continuing
5  n  fi  x  ,  ,..  |  left-to-right
6  n  fi  x  =  ≠  <  >  ≤  ≥  right-to-left
7  n  fi  x

The type and argument of a function must be on precedence level 0. Any data expression becomes
precedence level 0 by putting it in parentheses ( ). On level 6, the operators are “continuing”. This
means, for example, that a=b=c neither associates to the left (a=b)=c nor associates to the right
a=(b=c), but means (a=b)∧(b=c). Similarly a<b=c≤d means (a<b)∧(b=c)∧(c≤d). Whenever
“data” appears in an alternative for “program”, all forms of data are allowed, with this exception:
the argument of a plan must be on precedence level 0. Only one alternative for “data” contains
“program”, and there all forms of program are allowed.

**Data**

ProTem’s basic data are numbers, characters, binary values, and identities. ProTem’s data structures
are bunches, sets, strings, and lists. In addition, there are functions and result-expressions.

**Numbers**

Numbers are not divided into disjoint types. A natural number is an integer number; an integer
number is a rational number; a rational number is a real number; a real number is a complex
number. There is also an infinite number ∞ greater than all other numbers.

In addition to the number symbols, there are predefined names of numbers such as pi (the ratio of a
circle’s circumference to its diameter), e (the base of the natural logarithms), and i (the imaginary
unit, a square root of −1). Predefined names can be redefined. The postfix operator % means
division by 100; for examples, 99.9%, x% and (x+y)% . There are two 1-operand prefix
operators + and − . There are nine 2-operand infix operators + – × / ∧ ∧ ∧ ∨ & . There are
predefined function names such as abs, arc, arccos, arcsin, arctan, ceil, cos, cosh, div, exp, floor, im,
ln, log, mod, re, round, sin, sinh, sqrt, tan, and tanh (see Predefined Names). Division of integers,
such as 1/2, may produce a noninteger. Exponentiation is 2-operand infix ^; for example,
1.2*10^3 (one point two times ten to the power three), which can be written more briefly as
1.2^3. More generally, x^y = x×10^y . The operator ∧ is minimum (arms down, does not hold
water; note that ^ and ∧ are different). The operator ∨ is maximum (arms up, holds water). The
complex number x + i×y can be written more briefly as x&y.
Characters

A character is a text of length 1. We leave it to each implementation to list the characters, and to state their order. In addition to the character symbols such as “a” (small a) and “ ” (space), there are six predefined character names: delete (backspace), tab, nl (new line, next line, return, enter), click, doubleclick, and end (the end-of-file character). Predefined functions suc and pre give the successor and predecessor in the character order. Predefined functions charnat and natchar map between characters and their (possibly extended ASCII or unicode) numeric encodings. Character combinations, for example, shift-option-a, also have numeric encodings.

Binary Values

The two binary values are $\top$ and $\perp$. Negation is $\neg$, conjunction is $\land$, disjunction is $\lor$.

The infix 2-operand operators $\equiv$ and $\neq$ apply to all data in ProTem with a binary result; the two operands may even be of different types. The order operators $\lt \leq \gt \geq$ apply to real numbers (including rationals, integers, and naturals), to characters, to binary values, to sets (subset, superset), to strings of ordered items, and to lists of ordered items, with a binary result. In the number order, $-\infty$ is smallest and $\infty$ is largest. In the character order, “ ” (space) comes first and end comes last. In the binary order, $\perp$ is below $\top$, so $\leq$ is implication. The postfix operator $!!$ applies to channels, and has a binary result saying whether there is written but unread data on the channel.

Identities

Identity values are obtained from biometric scanners, or from password encoding (encrypting). The textid function maps a text to an identity. Identities can be named, used in data structures, assigned, and communicated. They are used in permits (see Permit and Session). They are used as signatures in documents.

Bunches

There are several predefined bunch names:

- null: the empty bunch
- nat: all natural numbers. Examples: 0, 1, 2
- int: all integer numbers. Examples: $-2, -1, 0, 1, 2$
- rat: all rational numbers. Example: $1/2$
- real: all real numbers. Example: $2^{1/2}$
- com: all complex numbers. Examples: $(-1)^{1/2}, 3&4$
- char: all characters. Example: “a”
- bin: both binary values: $\top, \perp$
- text: all texts (character strings). Example: “abc”
- everyone: all identities
- all: all ProTem items

Any number, character, binary value, identity, set, string of elements, and list of elements is an elementary bunch, or synonymously, an element. For example, the number 2 is an elementary bunch, or element. Every expression is a bunch expression, though not all are elementary.

Bunch union is denoted by a comma: $A, B$

For example,
2, 3, 5, 7
is a bunch of four integers. There is also the notation
\[ x..y \]
where \( x \) and \( y \) are integers or \( \infty \) or \( -\infty \) or characters that satisfy \( x \leq y \). Note that \( x \) is included and \( y \) is excluded. For example, \( 0..10 \) is a bunch consisting of the first ten natural numbers, and \( 5..5 \) is the empty bunch \( \text{null} \).

For any \( A \) and \( B \),
\[ A: B \]
is binary. The size (or cardinality) of \( A \) is \( \varepsilon A \). For examples, \( \varepsilon(0, 1) = 2 \) and \( \varepsilon \text{null} = 0 \) and \( \varepsilon(a..b) = b-a \).

Bunches are equal if and only if they consist of the same elements, ignoring order and multiplicity.

In ProTem, all operators whose precedence is before that of bunch union, except \( \varepsilon \) and \( \mathcal{I} \), distribute over bunch union. Infix \( * \) distributes in its left operand only. For examples,
\[ -(3, 5) = -3, -5 \]
\[ (2, 3)+(4, 5) = 6, 7, 8 \]
This makes it easy to express the plural naturals (\( \text{nat}+2 \)), the even naturals (\( \text{nat} \times 2 \)), the square naturals (\( \text{nat}^2 \)), the natural powers of two (\( 2^\text{nat} \)), and many other things.

Bunches serve as a type structure in ProTem, as the contents of sets, and other uses.

**Sets**

A set is formed by enclosing a bunch in set braces. For examples, \( \{0, 2, 5\} \), \( \{0..100\} \), \( \{\text{null}\} \), \( \{\text{nat}\} \). The inverse of set formation is the content operator \( ~ \). For example, \( ~\{0, 1\} = 0, 1 \). The size (or cardinality) of a set, traditionally written \( |S| \), is therefore \( \varepsilon~S \) in ProTem. For examples, \( \varepsilon~\{0, 1\} = 2 \) and \( \varepsilon~\{\text{null}\} = 0 \). The element relation, traditionally written \( x \in S \), is therefore \( x:~S \) in ProTem. The union operator, traditionally \( \cup \), is \( \lor \) in ProTem. The intersection operator, traditionally \( \cap \), is \( \land \). Subset, traditionally \( \subseteq \), is \( \leq \); strict subset is \( < \); superset is \( \geq \); strict superset is \( > \). The power operator \( \mathcal{I} \) takes a bunch as operand and produces all sets that contain only elements of the bunch. For example, \( \mathcal{I}(0, 1) = \{\text{null}\}, \{0\}, \{1\}, \{0, 1\} \).

**Strings**

There is a predefined string name:
\[ \text{nil} \]
the empty string

Any number, character, binary value, identity, list, and function is a one-item string, or synonymously, an item. For example, the number 2 is a one-item string, or item.

String join is denoted by a semi-colon:
\[ S; T \]
\[ S \text{ join } T \]
For example,
\[ 2; 3; 5; 7 \]
is a string of four integers. There is also the notation
\[ x..y \]
\( x \) to \( y \) (same pronunciation as \( x..y \))
where \( x \) and \( y \) are integers or characters that satisfy \( x \leq y \). Again, \( x \) is included and \( y \) is excluded. For examples, 0;..10 is a string consisting of the first ten natural numbers, and 5;..5 = \text{null}.
The length of a string is obtained by the ↔ operator. For examples, ↔(2; 3; 5; 7) = 4, and ↔(x;..;y) = y−x.

A string is indexed by the \ operator. Indexing is from 0. For example, (2; 3; 5; 7)\2 = 5. A string can be indexed by a string. For example, (3; 5; 7; 9)(2; 1; 2) = 7;5;7.

If S is a string and n is an index of S and i is any item, then S⊲n⊳i is a string like S except that item n is i. For example, 3; 5; 9 ⊲2⊳8 = 3; 5; 8. This operator associates from left to right, so 3; 5; 9 ⊲2⊳8 ⊲1⊳7 = ((3; 5; 9) ⊲2⊳8) ⊲1⊳7 = (3; 5; 8) ⊲1⊳7 = 3; 7; 8. And 3; 5; 9 ⊲2⊳8 ⊲2⊳7 = ((3; 5; 9) ⊲2⊳8) ⊲2⊳7 = (3; 5; 8) ⊲2⊳7 = 3; 5; 7.

A text is a more convenient notation for a string of characters.
“abc” = “a”; “b”; “c”
“He said “Hi”.’ = “H”; “e”; “” ; “s”; “a”; “i”; “d”; “” ; “” ; “H”; “i”; “” ; “”
“abcdefgij”\(3;..6) = “def”

Strings are equal if and only if they have the same length, and corresponding items are equal. They are ordered lexicographically. For examples,
3; 5 < 3; 5; 2 < 3; 6

A bunch of items is an item. Since string join precedes bunch union on the precedence table, we have
(3, 4); (5, 6) = 3;5, 3;6, 4;5, 4;6
A string is an element (elementary bunch) if and only if all its items are elements.

If S is a string and n is a natural number, then
n * S n copies of S or n S’s
is a string, and
*S strings of S or any number of S’s
is a bunch of strings. For examples,
3*5 = 5;5;5
3*(4, 5) = 4;4;4, 4;4;5, 4;5;4, 4;5;5 5;4;4, 5;4;5, 5;5;4, 5;5;5
*5 = nil, 5, 5;5, 5;5;5, 5;5;5;5, and so on
The * operator distributes over bunch union, but in its left operand only.
null*5 = null
(2,3)*5 = 2*5, 3*5 = 5;5, 5;5;5
Using this semi-distributivity, we have
*a = nat*a

Lists

A list is a packaged string. It can be written as a string enclosed in square brackets. For example,
[0; 1; 2]

The list operators are domain, content, indexing, pointer indexing, join, composition, selective union, and comparisons. Let L and M be lists, let n be a natural number, and let p be a string of natural numbers. The list operators are:
□ L domain of L
~ L content of L
# L length of L
List structures:

- \( L n \) is \( L \) at index \( n \)
- \( L @ p \) is \( L \) at pointer \( p \)
- \( L ; M \) is \( L \) join \( M \)
- \( L M \) is \( L \) composed with \( M \)
- \( L \mid M \) is \( L \) otherwise \( M \), the selective union of \( L \) and \( M \)
- \( i \rightarrow x \mid L \) is item \( x \) and otherwise \( L \)

plus the comparisons \( L = M \), \( L \neq M \), \( L < M \), \( L > M \), \( L \leq M \), \( L \geq M \). Here are some examples.

\[
\begin{align*}
\Box[10; 11; 12] &= 0, 1, 2 & \text{the domain of a list} \\
\sim[10; 11; 12] &= 10; 11; 12 & \text{the content of a list} \\
\#[10; 11; 12] &= 3 & \text{the length, or number of items, in a list} \\
[10;..20] 5 &= 15 & \text{indexing starts at zero} \\
\{[2; 3]; 4; [5; [6; 7]] \} @ (2; 1; 0) &= 6 & \text{joining lists} \\
[10;..20] [3; 6; 5] &= [13; 16; 15] & \text{composition} \ (L M)n = L(Mn)
\end{align*}
\]

By using the \( @ \) operator, a string acts as a pointer to select an item from within an irregular structure. If the list \( L \mid M \) is indexed with \( n \), the result is either \( L n \) or \( M n \) depending on whether \( n \) is in the domain \( (0,..\#L) \) of \( L \). If it is, the result is \( L n \), otherwise the result is \( M n \).

\[
\begin{align*}
[10; 11] | [0;..10] &= [10; 11; 2;..10] \\
1 \rightarrow 21 | [10; 11; 12] &= [10; 21; 12]
\end{align*}
\]

The index can be a string, as in

\[
\begin{align*}
(0;1) \rightarrow 6 | [[0; 1; 2]; \\
[3; 4; 5]] &= [[0; 6; 2]; \\
[3; 4; 5]]
\end{align*}
\]

When a string or list is indexed by a structure, the result has the same structure as the index. For examples

\[
\begin{align*}
(10;..20) \setminus [2; (3, 4); [5; [6; 7]]] &= [12; (13, 14); [15; [16; 17]]] \\
(10;..20) [2; (3, 4); [5; [6; 7]]] &= [12; (13, 14); [15; [16; 17]]]
\end{align*}
\]

Let \( S = 10; 11; 12 \). Then

\[
\begin{align*}
S(0, \{1, [2; 1]; 0\}) &= S0, \{S1, [S2; S1]; S0\} \\
&= 10, \{11, [12; 11]; 10\}
\end{align*}
\]

Let \( L = [10; 11; 12] \). Then

\[
\begin{align*}
L (0, \{1, [2; 1]; 0\}) &= L0, \{L 1, [L 2; L 1]; L 0\} \\
&= 10, \{11, [12; 11]; 10\}
\end{align*}
\]

An operator that does not apply to a list can be composed with a list. For example,

\[
- [3; 5; 2] = [-3; -5; -2]
\]

Lists are equal if and only if they are the same length and corresponding items are equal. They are ordered lexicographically.

\[
[3; 5] < [3; 5; 2] < [3; 6]
\]

The list brackets \([ \] \) distribute over bunch union. For example,

\[
[0, 1] = [0], [1]
\]

Thus \([10*\text{nat}]\) is all lists of length 10 whose items are natural, and \([4*[6*\text{real}]]\) is all 4 by 6 arrays of reals.
Conditional Data

The 3-operand expression $x \n v y \n w z$, pronounced “if $x$ then $y$ else $z$”, has binary operand $x$, but $y$ and $z$ are of arbitrary type. For example,

$$y \neq 0 \n v x / y \n w \text{ "nan"}$$

If $y \neq 0$ has value $\top$, then this data expression has number value $x / y$. If $y \neq 0$ has value $\bot$, then this data expression has text value “nan”. This operator associates from right to left so that it can be evaluated from left to right. For example,

$$(a \n v b \n w c \n v d \n w e) = (a \n v b \n w (c \n v d \n w e))$$

If $a$ has value $\top$, then this expression has value $b$, with no need to evaluate further.

Functions

A function defines a parameter; that is its only job. Let $p$ (parameter) be any simple name, let $D$ (domain) be any expression (but not using $p$), and let $B$ (body) be any expression (possibly using $p$ as a constant name for an element of $D$). Then $\langle p: D \to B \rangle$ is a function with parameter $p$, domain $D$, and body $B$. For example,

$$\langle n: \text{nat} \to n+1 \rangle \text{ map } n \text{ in \nat \to } n+1$$

is the successor function on the natural numbers. The parameter name begins its scope at $\langle$ and ends its scope at $\rangle$ (see Scope). The $\boxdot$ operator gives the domain of a function. For example,

$$\boxdot(\langle n: \text{nat} \to n+1 \rangle) = \text{nat}$$

The $\#$ operator gives the size of the function, which is the size of its domain. For example,

$$\#(\langle n: (0,..10) \to n+1 \rangle) = 10$$

A function of $n+1$ parameters is a function of 1 parameter whose body is a function of $n$ parameters. For example, the maximum function

$$\langle a: \text{real} \to \langle b: \text{real} \to a \n v b = a = b \rangle \rangle$$

has two parameters. The notation for applying a function to an argument is the same as that for indexing a list: adjacency. If $f$ is a function of two parameters, then $f x y$ applies $f$ to $x$ and $y$. Caution: in some languages, applying $f$ to $x$ and $y$ is $f(x,y)$. In ProTem, comma is bunch union, and function application distributes over bunch union. So in ProTem, $f(x,y) = f x, f y$.

The predefined function $\text{form}$ has four parameters. The first three parameters say how to format a number, and the last is the number to be formatted. For example, $\text{form} 4 1 10 pi = "3.1416^{^2}0"$. We can define a new function

$$\text{new myform} = \text{form} 4 1 10$$

by supplying just three parameters, and then apply it to a number to be formatted:

$$\text{myform} pi = "3.1416^{^2}0"$$

When the body of a function does not use its parameter, there is a syntax that omits the angle brackets $\langle \rangle$, unused name, and colon. For example,

$$2 \to 3$$

means $\langle n: 2 \to 3 \rangle$ or choose any other parameter name.

Allowing the body of a function to be a bunch generalizes the function to a relation. For example, $\text{nat} \to \text{bin}$ can be viewed in either of the following two ways: it is a function (with unused and therefore omitted parameter) that maps each natural to $\text{bin}$; it is all functions with domain at least $\text{nat}$ and range at most $\text{bin}$. As an example of the latter view, we have

$$\langle i: \text{int} \to \text{mod i 2 = 0} \rangle : \text{nat} \to \text{bin}$$

The function $\langle f: (\text{int} \to \text{int}) \to f 2 \rangle$ is “higher order”, which means it has a function-valued parameter.
It can be applied to any function with domain at least \textit{int} and range at most \textit{int}.

Let $f$ and $g$ be functions such that $f$ is not in the domain of $g$ \((\neg f \sqsubseteq g)\). Then $gf$ is the composition of $g$ and $f$.

\[(gf)x = g(fx)\]

If $f$ and $g$ are functions, then

\[f \upharpoonright g\]

is a function that behaves like $f$ when applied to an argument in the domain of $f$, and otherwise behaves like $g$.

\[
\square(f \upharpoonright g) = \square f, \square g \\
(f \upharpoonright g)x = (x; \square' x \neq g x)
\]

Composition and selective union can have any mixture of operators, lists, and functions as operands. For example,

\[-[3; 5; 2] = [-3; -5; -2] \\
even [3; 5; 2] = [\bot; \bot; \top] \\
(-even)x = -(even x)
\]

Argumentation comes before bunch union in precedence, and so it distributes over bunch union.

\[(f, g)(x, y) = fx, fy, gx, gy\]

If you want to apply a function to a bunch without distributing over the elements of the bunch, you must package the bunch as a set.

\textbf{result-Expressions}

A \textbf{result}-expression allows us to use a program to compute data. It has the form

\texttt{result} simplename : data := data \(\llbracket\) program \(\rrbracket\)

A local variable is defined with a type and initial value. Then the program is executed. The result is the final value of the newly defined local variable. We have not yet presented programs, but the following example, which approximates the base of the natural logarithms $e$, should give the idea.

\texttt{result} sum : \texttt{rat} := 1 \(\llbracket\)
\texttt{for} i := 1..15 \(\llbracket\)
\texttt{term} := term / i. sum := sum + term \(\rrbracket\)

There are no side effects. Nonlocal variables become constants within the program; their values may be used, but assigning them is not permitted. Input from and output to nonlocal channels are not permitted.

All the ways of expressing data can be combined arbitrarily, without restriction. Here is a function whose body is a \textbf{result}-expression. It expresses the number of times $2$ is a factor of $n$.

\texttt{result} \(\langle n: (\texttt{nat}+1) \rightarrow\) result \(f: 0..n:= 0\)
\(\llbracket\) \texttt{new} \(m: 1..n+1:= n.\)
\(\texttt{loop} \llbracket\) \texttt{if even m} \(f:= f+1.\) \texttt{m:= m/2.} \texttt{loop} \(\rrbracket\)\(\rrbracket\)

A \textbf{result}-variable begins its scope after \(\llbracket\) and ends its scope at the corresponding \(\rrbracket\) (see \texttt{Scope}). Consequently, the \textbf{result}-variable can be any simple name, even one that has already been defined in the scope that encloses the \textbf{result}-expression. The type and initial value of the \textbf{result}-variable cannot use the \textbf{result}-variable.


**Quote and Unquote**

The predefined function `quote` takes any data and produces a text representation of the data. Roughly speaking, it quotes its argument. For examples, `quote 123 = “123”`, `quote ⊤ = “⊤”`, `quote “abc” = “_abc_”`, `quote {0, [1; 2]} = “{0,[1;2]}”`. The argument is evaluated before quoting; for examples, `quote (2×3) = “6”`, `quote (0<1) = “⊤”`. In a context where `x=3`, `quote (x×4) = “12”`. Function `quote` does not provide any control over the format of the resulting text. For numbers, fine control over the format of the resulting text is provided by the predefined function `form`.

The predefined function `unquote` is a kind of inverse of `quote`. It applies to texts; roughly speaking, it unquotes its argument. For examples, `unquote “123” = 123`, `unquote “⊤” = ⊤`, `unquote ““abc”” = “abc”`, `unquote “{0, [1; 2]}” = {0, [1; 2]}`. The argument is evaluated after unquoting; for examples, `unquote “2×3” = 6`, `unquote “0<1” = ⊤`. In a context where `x=3`, `unquote “x×4” = 12`. If the argument does not represent a ProTem data expression, the result is “error”.

Whenever a data that is not a text is used in a context that requires a text, the `quote` function is applied automatically. For example, output to the screen, denoted `!`, requires a text. So `! 123` places a number where a text should be. So `quote` is applied automatically (`! quote 123`) resulting in a text (`! “123”`) as required for output. If `quote` has been redefined (see `Scope`), the predefined `quote` is used.

Whenever a text data is used in a context that requires a data that is not a text, the `unquote` function is applied automatically. For example,

```
“123” + 1 = unquote “123” + 1 = 123+1 = 124
```

If `unquote` has been redefined (see `Scope`), the predefined `unquote` is used. If the result is “error”, `unquote` is not further applied.

**Scope**

A simple name is defined in these six ways: by the keyword `new`, as a named program, as a function parameter, as a plan parameter, as a `for`-index, or as a `result`-variable. We shall come to each of these shortly. The scope of a simple name is the part of a program in which the name is defined. We shall also come to the ways of composing larger programs from smaller programs using program brackets `[ ]`. Scopes are limited by `[ ]`, and by `{ }`. The opening bracket `[` or `{` opens a scope, and the corresponding closing bracket `]` or `}` closes the scope.

Except for dictionary definition, a simple name defined in a local scope using the keyword `new` must be new, not already defined since the most recent scope opener `[`. Its scope extends from its definition through all following sequentially composed programs to the scope closer `]` corresponding to the most recent scope opener `[`. But it may be covered by a redefinition in an inner scope. Using `new x=2` and `new x=3` as example definitions, and letting `A`, `B`, `C`, `D`, and `E` stand for arbitrary program forms (but not `new` or `old`), in

```
[A. new x=2. B. [C. new x=3. D]]. E
```

the definition of `x` as the number 2 is not yet in effect in `A`, but it is in effect in `B`, `C`, and `E`. The definition that makes `x` the number 3 is in effect in `D`. None of `A`, `B`, `C`, `D`, or `E` can contain a redefinition of `x` unless it is within further scope limiters `[ ]` or `{ }`.

A name defined by `new` can become undefined by the keyword `old`, ending its scope early. So in
new \( x = 2 \). A. old \( x \). B

the definition of \( x \) is in effect in A but not in B. Within B, the name \( x \) has the same meaning (if any) that it had before the definition new \( x = 2 \). After old \( x \), the name \( x \) is again new and available for definition. However,

new \( x = 2 \). \( \text{old } x \). A

is not allowed; a scope cannot be ended by old within a subscope (except for dictionaries). Dictionary definitions work differently (see Dictionary Definition).

A scope can be nested inside another scope, which can be nested inside another, and so on. Outside all scope limiters is the persistent scope. A name defined by new in the persistent scope is called a persistent name. When you define a new name in the persistent scope, the name must differ from all names in the persistent scope for which you are on one or both of its permit-lists (see Permit). Its scope ends only with old. Its scope does not end with the end of a computing session, not even by switching off the power. Persistent variables serve as “files”. In the persistent scope, there is a dictionary named predefined that contains the predefined names.

Whenever a simple name is used, it is looked up as follows: first, look in the most local scope (innermost \( [ ] \) or \( \langle \rangle \)); then look in the next most local scope; then the next, and so on; when all local scopes have been searched, look in the persistent scope; then look in the predefined dictionary.

Whenever a compound name is used, the search is in the persistent scope, then in the predefined dictionary.

Programs

Some program constructs are concerned with names: creating a name (new), deleting a name (old). Other program constructs are variable assignment, input, output, and a variety of ways of combining programs to form larger programs. All programs, including those that create and delete names, are executed in their turn, just like variable assignments and input and output.

Variable Definition

Variable definition has the form

new newname : data := data

The newname becomes a variablename. Here is an example variable definition.

new \( x \) : nat := 5

This defines \( x \) to be a variable assignable to any element in nat, and initially assigned to 5. There is no such thing as an “uninitialized variable” nor the “undefined value” in ProTem. In a variable definition, the data after : is called the “type” of the variable, and the data after := is called the “initial value”. The type can be anything except the empty bunch, and the initial value must be an element of the type. The type and initial value can depend on previously defined names, including variables. For example,

new \( y \) : 0..2 \( x \) := \( x \)

defines \( y \) as a variable whose value can be any natural number from (including) 0 up to (excluding) twice the current value of \( x \) (the value of \( x \) at the time this definition is executed), with initial value equal to the current value of \( x \). But the type and initial value cannot make use of the name being defined.

Here are three more examples.
new s: [10*int]:= [10*0]
new t: text:= ""
new u: (0..20)*char:= “abc”

In the first example, \( s \) is defined as a variable that can be assigned to any list of ten integers, and is initially assigned to the list of ten zeroes. In the middle example, \( text \) is a predefined bunch equal to \( *char \), so \( t \) can be assigned to any text, and is initially assigned to the empty text. In the last example, \( u \) is defined as a variable that can be assigned to any text of length less than 20, and is initially assigned to the text “abc”.

**Assignment**

A variable can be reassigned by the assignment program. It has the form

\[ \text{variable name} := \text{data} \]

Here are two examples using the definitions of the previous subsection.

\[
\begin{align*}
x &:= x+1 \\
s &:= 3 \rightarrow 5 | s
\end{align*}
\]

The data on the right of \( := \) must be an element in the type of the variable on the left of \( := \). As in the examples, the data on the right of \( := \) can make use of the variable on the left of \( := \).

**Constant Definition**

Constant definition has the form

\[ \text{new newname} := \text{data} \]

The newname becomes a constantname. The data on the right of \( := \) cannot make use of the name on the left of \( := \). The newly created constantname cannot be reassigned. Here are three constant definitions.

\[
\begin{align*}
\text{new size} &:= 10 \\
\text{new piBy2} &:= pi / 2 \\
\text{new range} &:= 0..size
\end{align*}
\]

where \( pi \) is a predefined constant name.

A constant may use variables to express its value. For example

\[ \text{new xplusI} := x+1 \]

The current value of variable \( x \) is used to evaluate \( x+1 \), and \( xplusI \) expresses that value. Variable \( x \) may later be reassigned to another value, but that does not affect the value of \( xplusI \). For example,

\[
\begin{align*}
\text{new x: nat} &:= 3. \quad \text{`x has value 3} \\
\text{new xplusI} &:= x+1. \quad \text{`x has value 3 and xplusI has value 4} \\
x &:= 5. \quad \text{`x has value 5 and xplusI has value 4}
\end{align*}
\]

**Data Definition**

Data definition has the form

\[ \text{new newname} = \text{data} \]

The newname becomes a dataname (note \( = \) rather than \( := \) as in constant definitions).

\[ \text{new xplus2} = x+2 \]

makes the value of \( xplus2 \) depend on the value of variable \( x \). As \( x \) changes value, \( xplus2 \) changes value so that \( xplus2 = x+2 \) is always \( \top \). In the constant definition of \( xplusI \) earlier, \( x+1 \) is evaluated once, at definition time. By contrast, in the data definition of \( xplus2 \), \( x+2 \) is not evaluated at definition time; it is evaluated every time \( xplus2 \) is used in a context that requires its value. For example,
new x: nat:= 3.
new xplus2 = x+2.
! xplus2. ` prints 5
x:= 6.
! xplus2 ` prints 8

A data definition can depend indirectly on a variable. For example,

new twoxplus4 = 2×xplus2

makes twoxplus4 depend indirectly on the value of variable x. In this context, the value of xplus2 is not required, so it is not evaluated. If x currently has value 3, then ! twoxplus4 prints 10. If x currently has value 6, then ! twoxplus4 prints 16.

Data Recursion

In a variable definition, the type and initial value cannot depend on the variable being defined.

new bad: 0..2×bad:= bad ` illegal

is not allowed due to the two occurrences of bad to the right of the colon. Likewise a constant definition cannot be recursive.

Data definition does allow recursion. The next two examples define fact and div to be the factorial function and integer division function for natural numbers.

new fact = 0 → 1 |〈n: (nat+1) → n×fact(n–1)〉

new div = 〈a: nat → 〈d: (nat+1) →
     a<d ⊨ 0 ⊨ even a ⊨ 2×div(a/2) d ⊨ 1 + div(a–d) d〉〉

Here is a bunch of texts (a grammar). This bunch includes the text “a+b+a–a”, and many more.

new term = “a”, “b”, term; “+”; term; “–”; term

This recursive definition is equivalent to the nonrecursive definition

new term = (“a”, “b”); *((“+”, “–”); (“a”, “b”))

Here is a function that eats arguments until it is fed argument 0.

new eat = 〈n: nat → n=0 ⊨ 0 ⊨ eat〉

So eat 5 2 0 = 0, and eat 4 7 3 8 0 = 0, and eat 1 2 = eat.

The next example defines all binary trees with integer nodes.

new tree = [nil], [tree; int; tree]

The final example is a pure, baseless recursion.

new rec = rec

Whenever rec is used, its evaluation is nonterminating.

Constant v Data Definition

A constant definition evaluates its data once, at definition time, whereas a data definition evaluates its data each time its value is required. If the data is fully evaluated, there is no difference. For example, there is no difference between these two definitions:

new five:= 5
new five = 5

When there are no variables used to express the value (neither directly nor indirectly), there is no
semantic difference between data definition and constant definition, but there may be an efficiency difference. Compare these two definitions.

\[
\text{new } \textit{six} := 5+1 \\
\text{new } \textit{six} = 5+1 
\]

If the value of \textit{six} is never required, the data definition (=) is more efficient. If the value of \textit{six} is required once, they are equally efficient. If the value of \textit{six} is required two or more times, the constant definition ( := ) is more efficient. Here is a more interesting comparison.

\[
\text{new } \textit{double} := \langle n: (0..10) \rightarrow 2\times n \rangle \\
\text{new } \textit{double} = \langle n: (0..10) \rightarrow 2\times n \rangle 
\]

The constant definition causes the function to be evaluated by applying it to all its arguments and storing the results. In effect, the function is evaluated to the list \([0; 2; 4; 6; 8; 10; 12; 14; 16; 18]\).

Then, when the value of \textit{double} applied to an argument is required, that argument indexes the list. The data definition does not evaluate the function. Each time the value of \textit{double} applied to an argument is required, the body of the function is evaluated. Which one is more efficient depends on the size of the domain, the complexity of the result, and the number of times the definition is used.

**Program Definition**

Program definition has the form

\[
\text{new } \textit{name} \[ \text{program} \] 
\]

The \textit{name} becomes a programname. Program definition gives a program a name, but does not execute the program. For example,

\[
\text{new } \textit{switchends} \[ \textit{s} := 0 \rightarrow s 9 | 9 \rightarrow s 0 | s \] 
\]

Execution of this definition creates the program name \textit{switchends}, but does not execute program \[ \textit{s} := 0 \rightarrow s 9 | 9 \rightarrow s 0 | s \]. After execution of this definition, the name \textit{switchends} can be used to call (cause execution of) the program it names. Program definitions can be recursive. Predefined program names include \textit{await}, \textit{exec}, \textit{ok}, \textit{stop}, \textit{wait}.

A fancy name can be used as a specification. For example,

\[
\text{new } \langle x' > x \rangle \[ x := x+1 \] 
\]

The specification \langle x' > x \rangle is implemented (refined, implied) by the program \[ x := x+1 \]. A prover is invoked by the [esc v] command (see Verify). If the specification is written within the language that the prover understands, the prover attempts to prove that the specification is implemented (refined, implied) by the program. If the program makes use of a specification, the inner specification is used in the outer proof. For example,

\[
\text{new } \langle x' = 0 \rangle \[ \text{if } x \neq 0 \[ x := x-1. \langle x' = 0 \rangle \] ] 
\]

In the program, the specification \langle x' = 0 \rangle means exactly what it says, rather than the program that it names. Thus the use of specifications makes complicated fixed-point semantics unnecessary. If the prover fails to understand the specification, or fails to prove the refinement, it informs the programmer, and treats the specification as just a name. (See the paper Specified Blocks.)

**Measuring Unit Definition**

There are three predefined units of measurement. They are \textit{g}, representing mass in grams, \textit{m}, representing distance in meters, and \textit{s}, representing time in seconds. A unit of measurement has all the properties of an unknown positive finite real number constant. So, for example, we write \textit{10 m/s} for the speed 10 meters per second. And we can define

\[
\text{new } \textit{km} := 1000 \times \textit{m} 
\]

to make \textit{km} be a kilometer, and
\textbf{new} \texttt{h} := 3600 \times s \\

\text{to make \texttt{h} be an hour. So} \ 1 \text{m/s} = 3.6 \text{km/h} \ \text{evaluates to} \ T. \ \text{To assign a variable to a quantity with units attached, the variable's type must have compatible units attached. For example,}

\textbf{new} \ \texttt{speed: real} \times \texttt{m/s} := 3.6 \times \texttt{km/h}

\text{assigns} \ \texttt{speed} \ \text{to} \ 1 \times \texttt{m/s}.

You can create a new unit of measurement, unrelated to the existing units. Measuring unit definition has the form

\textbf{new} \ \texttt{newname} #1

The \texttt{newname} becomes a unitname. For example,

\textbf{new} \ \texttt{sheet} #1

creates a new unit of measurement called the \texttt{sheet}. Now you can define the related units

\textbf{new} \ \texttt{quire} := 25 \times \texttt{sheet}  \\
\textbf{new} \ \texttt{ream} := 20 \times \texttt{quire}

Now you can define a variable using the new units.

\textbf{new} \ \texttt{order: nat} \times \texttt{sheet} := 3 \times \texttt{ream}

This assigns \texttt{order} to \ 1500 \times \texttt{sheet}.

When the value \ 5 \times \texttt{m/s} \ is converted to text by \texttt{quote}, the result is “5 \text{m/s}” without the \times \ sign and without evaluating the unknown real value \texttt{m/s}. And \texttt{unquote} “5 \text{m/s}” = 5 \times \texttt{m/s}. Similarly for all units of measurement. One more example: \texttt{quote} (2 \times 3 \times \texttt{km/h}) = “1.6667 \text{m/s}”.

\textbf{Forward Definition}

Forward definition has the form

\textbf{new} \ \texttt{newname}

The \texttt{newname} becomes either a dataname or a programname. For example

\textbf{new} \ \texttt{abc}

is a notice that a definition will follow later in the same scope. In a data definition or program definition, the scope of the name being defined starts immediately. A forward definition allows mutual recursion by starting the scope of a data name or program name even before its definition. For example, using \cdots \ to stand for uninteresting things, in

\textbf{new} \ \texttt{f} := 3. \ [\textbf{[new} \ \texttt{f. new} \ \texttt{g} = \cdots \texttt{f} \cdots \texttt{g} \cdots. \ \textbf{new} \ \texttt{f} = \cdots \texttt{f} \cdots \texttt{g} \cdots. \ \cdots]\]

the inner \texttt{f} and \texttt{g} are each defined in terms of both of them. Without the forward definition of \texttt{f} (following \[]), \texttt{g} \ would be defined in terms of the earlier constant definition \textbf{new} \ \texttt{f} := 3.

\textbf{Name Removal}

Names defined with the keyword \textbf{new} can be undefined with the keyword \textbf{old}. Name removal has the form

\textbf{old} \ \texttt{oldname}

Ironically, by saying \textbf{old} \ \texttt{x}, the name \texttt{x} becomes available for reuse as a new name. Even though a name becomes undefined, what it named will remain as long as there is an indirect way to refer to it. For example,

\textbf{new} \ \texttt{s: *all} := \texttt{nil.}

\textbf{new} \ \texttt{push} \ [\textbf{plan} \ \texttt{x: all} [\texttt{s} := \texttt{s};\texttt{x}]].

\textbf{new} \ \texttt{pop} \ [\texttt{s} := \texttt{s}(0;\leftrightarrow\texttt{s}–1)].

\textbf{new} \ \texttt{top} = \texttt{s}(\leftrightarrow\texttt{s}–1).

\textbf{new} \ \texttt{empty} = \texttt{s} = \texttt{nil.}

\textbf{old} \ \texttt{s}
The names *push*, *pop*, *top*, and *empty* are now defined and ready for use. The name *s* was defined for the purpose of defining the other names, and then removed, leaving the other names dependent upon an anonymous variable.

The predefined names include *randNat*, *randNatInit*, and *randNatNext*. They might have been defined as:

```plaintext
new big := 2^31.
new rv := 0..big := 123456789.
new randNat = (from: nat -> (to: nat -> floor (from + (to-from)rv/big))).
new randNatInit [plan seed: 0..big [rv := seed]].
new randNatNext [rv := mod (rv × 5^13) big].
```

The following sequence swaps the data names *i* and *j*.

```plaintext
new t = i. old i.
new i = j. old j.
new j = t. old t
```

**Output and Input**

Each channel is defined to transmit a specific type of value. The output channels *screen* and *printer*, and the input channel *keys* are predefined to transmit text. The input channel *microphone* and the output channel *speaker* are predefined to transmit sound. Input channel *time* transmits times. We can define local channels to transmit any type of value (see Channel Definition).

Output has the form

```plaintext
channelname ! data
```

Channel *screen* accepts text, which is displayed on the screen. The program

```plaintext
screen! "Hi there."
```

sends the text “Hi there.” to the screen. Output is buffered so it will be available when *screen* is ready to receive it. Texts can be joined and sent together.

```plaintext
screen! "Answer = "; quote x; nl
```

where *quote* is a predefined function that converts to a text, and *nl* is the new line character, or next line character, or return character. Function *quote* can be omitted (see Quote and Unquote).

When *screen* or *printer* receives the *delete* (backspace) character, the last previously sent character is deleted; when the *tab* character is received, some number of spaces are substituted; when the *nl* character is received, further characters start on a new line.

The keyboard is a program that runs concurrently with other programs; you don't need to initiate it; it is already running. It monitors what key combinations are pressed, and for what duration, and creates a string of characters. The shift-A combination is a single character “A”. Likewise the control-Q combination is a single character. The click button is just a key like any other; *click* is a character, and *doubleclick* is a character.

Input has two forms: without echo, and with echo. The first form, without echo, is

```plaintext
channelname ? data
```

Text from the keyboard (including the click button) can be received from channel *keys*. Five
characters of input are received from channel `keys` by saying

```
keys? 5*char
```

What follows `?` is called the pattern (or grammar). If input is not yet available, it is awaited. The input read is the earliest input on that channel that has not yet been read. The `tab`, `delete`, and `nl` characters may be part of the input; no corrections are made. The input is not echoed on the screen.

The shortest input that fits the pattern is read. The program

```
keys? text; nl
```

reads text up to and including the first `nl` character, but

```
keys? text
```

just inputs the empty text.

Patterns `digit` and `numpat` are prede fined as follows.

```
new digit := "0", "1", "2", "3", "4", "5", "6", "7", "8", "9".
new numpat := ("+", "–", "") ; digit; *digit; (("." ; digit; *digit), "")
```

To receive a text that can be interpreted as a number, possibly preceded or followed by spaces, possibly preceded by a sign, ending in a new line character, input

```
keys? *" "; numpat; *" "; nl
```

Without `nl`, leading spaces and an optional sign and the first digit are read.

When input is received, it is referred to by the channel name followed by `??`. After the previous example input, we might have the assignment

```
x := unquote (keys??)
```

where `unquote` is a predefined function that converts from a text to the type of variable `x`. Function `unquote` can be omitted (see `Quote` and `Unquote`).

If `c` is the name of an input channel, then `c!!` is a binary expression with value `⊤` if there is written but unread data on the channel, and `⊥` if there is not. For example,

```
if keys!! [keys? char; screen! keys??] else [screen! “Are you still there?”]
```

Input on a channel that does not currently have any written-but-not-yet-read data waits until data is written to the channel by a concurrent program.

If channel `c` is defined to input text, the program

```
c? “y”, “n”
```

inputs one character, either “y” or “n”, from channel `c`. If the first available character on channel `c` is “à”, or more generally, if the input on the channel does not fit the pattern, what happens is undefined. Here are three options.

- The program cannot be executed, so execution ends.
- An error message is sent to channel `screen` to say that the input is unacceptable, and execution ends.
- An error message is sent to channel `screen` to say that the input is unacceptable, and the sender is given another opportunity to send an input that fits the pattern.

What happens depends on the implementation and on the channel. Perhaps the last option is appropriate for channel `keys`, and the first is appropriate for a secure channel.

An input program consisting of an input channel name, a question mark, and a pattern, does not echo the input on the screen; the input is invisible. This is useful, for example, when reading a password (see `Read Password`). To input and echo together, character by character, add an exclamation mark and the output channel name for the echo.

```
channelname ? data ! channelname
```

For example,
keys? text; nl !screen

This program inputs, from channel keys, text to and including the first nl character, and outputs the same on channel screen. Each character is echoed as it is input. A space ←, tab ⇨, new line ↵, or delete character → is displayed visibly as a graphic symbol.

In an input with echo, the pattern between the input question mark and the echo exclamation mark can be omitted. This results in a special pattern called the correcting pattern. For example,

keys?!screen

The correcting pattern reads a line of text to and including the first nl character, but this text is corrected according to delete (backspace) characters. The nl character is consumed, but not included in the value read. After this input, the value of keys?? is the corrected text, not including the final nl. If, during correction, there are more delete characters than other characters, the extra delete characters are ignored. The echo displays space, tab, and new line characters as spaces, tabs, and new lines, and displays delete characters as deletions of previous characters.

An output channel name can be omitted, in which case the output channel is screen. For example,

! “Hello World”

prints Hello World on the screen. An input channel name can be omitted, in which case the input channel is keys. For example,

? char

reads one character from keys, with no echo. The most recent text read on channel keys can be referred to as just ?? . And !! is a binary expression saying whether there is written but unread data on channel keys. If the input channel is omitted, and the name keys has been redefined, the input channel is the predefined channel keys. If the output channel is omitted, and the name screen has been redefined, the output channel is the predefined channel screen.

In a context requiring a number, the expression unquote (keys??) can be written unquote (??) or keys?? or ?? . But it cannot be written unquote keys?? because that is parsed as (unquote keys)?? . And it cannot be written unquote ?? because the compiler will complain that unquote is not a channel. Similarly for quote (keys!!) .

The most common form of input

?! reads one line from keys, correcting it according to delete characters, up to the first nl, which is not included in the value of ??, and echoes character by character to screen.

In summary, output and input are, respectively,

channelname ! data
channelname ? pattern ! channelname

After reading input, the input most recently read is referred to as

channelname ??

The binary expression

channelname !!
says whether there is written but unread data on the channel. If the channelname is keys or screen it can be omitted. If input echoing is not wanted, omit both ! and the echo channelname. In an input program with echo, omitting the pattern results in the correcting pattern.
Sequential Composition

Sequential composition is denoted by a period (point, dot). According to the grammar, it is an infix connective; in other words, the period comes between and joins two programs.

\[
\text{program . program}
\]

In the persistent scope, each program is executed as soon as it is keyed in. The end of the sequence of keystrokes comprising a program to be executed is recognized by the period that will join it to the sequentially next program, after execution of the just completed program. So, in the persistent scope, the period feels more like a program terminator than a program joiner.

ProTem can be used as a calculator. In the persistent scope, the program

\[
! 2+2.
\]

with a following period and new line character, immediately prints \(4\). In full, it is

\[
\text{screen! quote (2+2)}.
\]

The program

\[
! 2+2
\]

followed by a new line character but without a period, is not executed until a period and another new line character are entered. The program and period and new line

\[
\text{new temp := 2+2.}
\]

saves the result of the calculation under the name \(temp\), perhaps for use in further calculation. The name \(temp\) persists from session to session until it is ended by \(\text{old temp}\). The program and period and new line

\[
\text{new myfiles_.}
\]

immediately creates a dictionary within which programs and data can be stored and edited and used. The program and following period and comment and new line character

\[
\text{new myfiles_prog [! “2+2=”.}
\]

\[
! 2+2]. \text{`this is a comment}
\]

immediately saves a new program named \(prog\) in existing dictionary \(myfiles\). The saved program is not immediately executed. The first period comes between two output programs, joining them. There is no period following the last of these two output programs. The new line following the first period does not indicate the completion of the program definition. It does indicate that the part of the program definition that came before the new line character is no longer correctable by \(\text{delete}\) (backspace) characters. The period at the end indicates the completion of the program definition, but the second line of program definition remains correctable by \(\text{delete}\) characters until a new line character following the comment is typed. Further corrections can be made using the editor command [\text{esc c]}(see \text{Edit}).

Concurrent Composition

Concurrent composition has the form

\[
\text{program \| program}
\]

The concurrent composition of programs \(P\), \(Q\), and \(R\) is \(P\|Q\|R\). A variable defined before the concurrent composition remains a variable in at most one of the programs in the concurrent composition; in all the other programs of the concurrent composition, it becomes a constant. For example,

\[
\]

In the second concurrent composition, variable \(a\) can be reassigned in one of the concurrent programs, but not in both; it is reassigned in the left program. Likewise variable \(b\) can be reassigned in one of the concurrent programs, but not in both; it is reassigned in the right program. At the start of \(A\), variable \(a\) has value 4, constant \(b\) has value 2, and data \(c\) has value 6. At
the start of $B$, constant $a$ has value 1, variable $b$ has value 8, and data $c$ has value 9. If $A$ does not reassign $a$, and $B$ does not reassign $b$, then at the start of $C$, variable $a$ has value 4, variable $b$ has value 8, and data $c$ has value 12. Concurrent programs cannot affect each other through assignments of variables. For co-operation, programs can communicate with each other on channels defined for the purpose (see Channel Definition).

**Channel Definition**

Channel definition has the form

```
new newname ? data ! data
```

The `newname` becomes a channelname. The definition

```
new c? nat! 0
```

defines $c$ to be a new local channel that transmits values of type $nat$, with 0 as initial output and input. It can be used for output and input. Now $c$?? refers to the most recent input on the channel, and $c$!! is a binary expression saying whether there is written but unread data on channel $c$. Before there has been any output or input, $c$?? refers to the initial output and input supplied in the channel definition, and $c$!! is $\bot$. The type of the channel cannot use the name of the channel being defined. Only one of the programs that are concurrent with each other can use a channel for output. More than one of the concurrent programs can use the same channel for input only if the concurrent composition is not sequentially followed by a program that uses that channel for input. When concurrent programs read from the same channel, they read the same inputs independently.

```
new c? nat! 0. x:= c?? `assigns x to 0
new c? nat! 0. c! 7. x:= c?? `assigns x to 0
new c? nat! 0. c! 7. c? nat. x:= c?? `assigns x to 7
new c? nat! 0. c? nat. c! 7. x:= c?? `deadlock, stuck at c? nat
new c? nat! 0. [c? nat. x:= c??] || c! 7 `assigns x to 7
new c? nat! 0. c! 7. c? nat. x:= c?? `assigns x to 7
```

**if-Program**

An if-program has the form

```
if data [ program ]
```

The if-program

```
if b [P]
```

is executed as follows: binary expression $b$ is evaluated; if its value is $\top$, then program $[P]$ is executed; if its value is $\bot$, then program $[P]$ is not executed. An if-else-program has the form

```
if data [ program ] else [ program ]
```

The if-else-program

```
if b [P] else [Q]
```

is executed as follows: binary expression $b$ is evaluated; if its value is $\top$, then program $[P]$ is executed and program $[Q]$ is not executed; if its value is $\bot$, then program $[P]$ is not executed and program $[Q]$ is executed.

**case-Program**

A case-program has the form

```
case data [ program ]
```

For example, the case-program with 3 cases,

```
case n [P, Q, R]
```

is executed as follows: natural expression $n$ is evaluated; then one of the sequence of programs
within the [ ] brackets is executed. (The periods between \( P \) and \( Q \), and between \( Q \) and \( R \), do not indicate sequential execution.) These programs, called cases, are numbered in order 0, 1, 2, and so on, and each is in a new scope. In the example, case 0 is \([P]\), case 1 is \([Q]\), and case 2 is \([R]\). In the example, if \( n \) has value 1, then just \([Q]\) is executed. If \( n \) is equal to or greater than the number of cases, an error message is printed and execution stops. The example case-program is equivalent to

\[
\text{if } n=0 [P] \text{ else if } n=1 [Q] \text{ else if } n=2 [R] \text{ else } [!] \text{"error: case index too large". stop}]
\]

It is allowed, but senseless, for any of the cases to be just a simple name definition. For example, if \( n \) is 1, and \( Q \) is \textbf{new} \( x=2 \), then \([\textbf{new} \ x=2]\) is executed, defining \( x \) and then immediately ending the scope of \( x \). Typically, the structure is something like

\[
\text{case } \ldots [\ldots]. [\ldots]. [\ldots]. [\ldots]
\]

so that each case can include useful definitions and a sequence of programs.

A case-else-program has the form

\[
\text{case } \textbf{data [ program ] else [ program ]}
\]

For example, the case-else-program with 3 cases,

\[
\text{case } n [P, Q, R] \text{ else } [S]
\]

is the same as the case-program, but if \( n \) is equal to or greater than the number of programs before else, then the program \([S]\) after else is executed. The example case-else-program is equivalent to

\[
\text{if } n=0 [P] \text{ else if } n=1 [Q] \text{ else if } n=2 [R] \text{ else } [S]]
\]

**for-Program**

A for-program has the form

\[
\text{for } \textbf{simplename := data [ program ]}
\]

The simplename becomes a constantname. Here is a nest of for-programs, or for-loops, that computes the transitive closure of \( A: [n^*[n^*\text{bin}]] \).

\[
\text{for } j:= 0;..n
\]

\[
[\text{for } i:= 0;..n
\]

\[
[\text{for } k:= 0;..n
\]

\[
[\text{if } A i j \land A j k
\]

\[
[A:= (i;k) \rightarrow \top \mid A]]]]
\]

The if-program \( A i j \land A j k \ [A:= (i;k) \rightarrow \top \mid A] \) can be restated as

\[
A:= (i;k) \rightarrow (A i k \lor (A i j \land A j k)) \mid A
\]

if you prefer. The name being defined by for is known only within the loop body, and it is known there as a constant, and so it is not assignable. It is called a for-index. In the example, each of the for-indexes takes values 0, 1, 2, and so on up to but not including \( n \).

For a second example, here is the sieve of Eratosthenes.

\[
\text{new } n:= 1000.
\]

\[
\text{new } \textbf{prime: } n^*\text{bin}:= 2^*\bot; (n-2)^*\top.
\]

\[
\text{for } i:= 2;..\text{ceil} (\text{sqrt } n) \ [\text{if } \textbf{prime} \ni [\text{for } j:= i;..\text{ceil} (n/i) \ [\textbf{prime}:= \textbf{prime} <i x j> \bot]]]]
\]

A for-index is “by initial value”, so

\[
\text{for } i:= x; x \ [x:= i+1]
\]

increases \( x \) by 1, not 2.

This next example prints the natural numbers forever.

\[
\text{for } n:= 0;..\infty \ [!] n; \ ""
\]

After the := we can have any string expression; the index stands for each item in the string, in
sequence. We can also have any bunch expression; the index stands for each element of the bunch, concurrently. As an example (note the use of \( \ldots \) rather than ;\( \ldots \) as earlier),

\[
\text{for } i := 0, \ldots \#L \ [L := i \rightarrow 0 \ | \ L]
\]

makes the items of \( \ L \) be 0 , concurrently. We could also write either of these:

\[
\text{for } i := \Box \ L \ [L := i \rightarrow 0 \ | \ L]
\]

\[
L := [\#L \ast 0]
\]

The domain of the \texttt{for}-index can also be a bunch of strings, or a string of bunches, and so on, so that sequential and concurrent execution can be nested within each other. (Note: distribution and factoring laws are not applied; the structure of the expression is the structure of execution.)

A \texttt{for}-index begins its scope after \( \[ \) and ends its scope at the corresponding \( \] \). Consequently, the \texttt{for}-index can be any simple name, even one that has already been defined in the scope that encloses the \texttt{for}-loop. The domain of the \texttt{for}-index cannot use the \texttt{for}-index.

**Named Program**

A named program has the form

\[
\text{simplename } [ \text{ program }]
\]

The simplename becomes a programname within the program that it names. It begins its scope after \( \[ \) and ends its scope at the corresponding \( \] \). Consequently, the name can be any simple name, even one that has already been defined in the scope that encloses the named program. The name is attached to the program (like a program definition), and the program is executed (unlike a program definition). One purpose of this naming is to make loops. Here is a two-dimensional search for \( x \) in an \( n \times m \) array \( A \) of integers (that is, \( A : [n \ast [m \ast \text{int}]] \)).

\[
\text{new } i : \text{nat} := 0.
\]

\[
\text{tryThisI } [\text{if } i = n \ [! x; \ “ \ does \ not \ occur.”]\]
\]

\[
\text{else } [\text{new } j : \text{nat} := 0.
\]

\[
\text{tryThisJ } [\text{if } j = m \ [i := i + 1. \ \text{tryThisI}]
\]

\[
\text{else } [\text{if } A i j = x \ [! x; \ “ \ occurs \ at ” ; i ; “ ” ; j]
\]

\[
\text{else } [j := j + 1. \ \text{tryThisJ}]])])]
\]

The next example is a fast remainder program, assigning natural variable \( r \) to the remainder when natural \( a \) is divided by positive natural \( d \), using only addition and subtraction.

\[
r := a.
\]

\[
\text{outerloop } [\text{if } r \geq d \ [\text{new } dd : \text{nat} := d.
\]

\[
\text{innerloop } [r := r – dd. \ dd := dd+dd.
\]

\[
\text{if } r < dd [\text{outerloop} \ \text{else } [\text{innerloop}]])])]
\]

The use of a program name is semantically a call; it means the same as replacing it with the program it names (including the \( [\ ] \) brackets). The fast remainder example means the same as

\[
r := a.
\]

\[
\text{outerloop } [\text{if } r \geq d \ [\text{new } dd : \text{nat} := d.
\]

\[
\text{innerloop } [r := r – dd. \ dd := dd+dd.
\]

\[
\text{if } r < dd [\text{if } r \geq d \ [\text{new } dd : \text{nat} := d.
\]

\[
\text{innerloop } [r := r – dd. \ dd := dd+dd.
\]

\[
\text{if } r < dd [\text{outerloop} \ \text{else } [\text{innerloop}]])])]
\]

\[
\text{else } [ [r := r – dd. \ dd := dd+dd.
\]

\[
\text{if } r < dd [\text{outerloop} \ \text{else } [\text{innerloop}]])])]
\]
The calls *outerloop* and *innerloop* were replaced by the programs they name. They reappear, and again they mean the programs they name. Although semantically they are calls, in the previous two examples they are last actions (tail recursions), so they are implemented as branches (jumps, go to’s).

The next example illustrates that named programs provide general recursion, not just tail recursion. It computes the Fibonacci numbers \( x := \text{fib} \, n \) and \( y := \text{fib} \,(n+1) \) in \( \log n \) time.

\[
\text{Fib} \, \begin{cases}
\text{if} \, n = 0 & [x := 0. \quad y := 1] \\
\text{else} & \begin{cases}
\text{if} \, \text{odd} \, n & [n := (n-1)/2. \quad \text{Fib} . \quad n := x. \quad x := x^2 + y^2. \quad y := 2nx + y^2] \\
\text{else} & [n := n/2 - 1. \quad \text{Fib} . \quad n := x. \quad x := 2nxy + y^2. \quad y := n^2 + y^2 + x] 
\end{cases}
\end{cases}
\]

As in a program definition, a fancy name can be used as a specification. For example,

\[
\begin{cases}
\text{odd} \, n \quad \text{and} \quad x \quad \text{even} \, \text{and} \quad y \quad \text{real}
\end{cases}
\]

The specification \( \langle \text{odd} \, n \rangle \) is implemented (refined, implied) by the program \( [x := x+1] \). A prover is invoked by the \texttt{esc v} command (see \texttt{Verify}). If the specification is written within the language that the prover understands, the prover attempts to prove that the specification is implemented (refined, implied) by the program. If the program makes use of a specification, the inner specification is used in the outer proof. For example,

\[
\begin{cases}
\text{odd} \, n \quad \text{and} \quad x > 0 \quad \text{and} \quad x < 0
\end{cases}
\]

In the program, the specification \( \langle x' > x \rangle \) means exactly what it says, rather than the program that it names. Thus the use of specifications makes complicated fixed-point semantics unnecessary. If the prover fails to understand the specification, or fails to prove the refinement, it informs the programmer, and treats the specification as just a name. (For more on proving, see the paper \texttt{Specified Blocks}.)

Suppose a name is defined within a loop. For example, the name \( a \) in

\[
\text{infiniteLoop} \, \begin{cases}
\text{new} \, a := \text{“a”}. \quad \text{la.} \quad \text{infiniteLoop}
\end{cases}
\]

Executing this loop prints an infinite sequence of the letter “a”. Replacing the call with the called program, it is equivalent to

\[
\text{infiniteLoop} \, \begin{cases}
\text{new} \, a := \text{“a”}. \quad \text{la.} \quad [\text{new} \, a := \text{“a”}. \quad \text{la.} \quad \text{infiniteLoop}]
\end{cases}
\]

In a general recursion, each call opens a new scope, and each new definition hides but does not destroy the previous definition. But when the recursive call is the last action performed in the named program (a tail recursion), as in this example, the old scope and its definitions cannot be used again, so the new scope replaces the old one; the scopes and variables do not pile up.

Let \( name \) be a new name (not defined in the local scope), and let \( program \) be a program, possibly using the name \( name \). Then the following three lines are equivalent to each other.

\[
\begin{cases}
\text{name} \quad \text{[program]}
\end{cases}
\]

\[
\begin{cases}
\text{new} \, \text{name} \quad \text{[program]. name}
\end{cases}
\]

\[
\begin{cases}
\text{new} \, \text{name} \quad \text{[program]. name. old name}
\end{cases}
\]

**Plan**

A plan is a program with a parameter. There are four forms of plan. The first is

\[
\text{plan} \quad \text{simplename : data [ [ program ] ]}
\]

The simplename is being introduced as a parameter within the program. It can be any simple name, even one that has already been defined in the current scope. Its type (after the \( : \)) cannot make use of the parameter. The scope of the parameter is from \( [ [ \text{to} ] ] \). For example,

\[
\text{plan} \, y : \text{real} \, [ x := xxy ]
\]

A plan can be argumented in the same way that lists are indexed and functions are argumented. The argument provides a value for the parameter. For example,
\[
\text{plan } y: \text{real } \{ x := x y \} 3
\]
is the same as
\[
x := x \times 3
\]

A program with \( n+1 \) parameters is a program with 1 parameter whose body is a program with \( n \) parameters. For example, here is a program with two parameters, followed by two arguments.

\[
\text{plan } x: \text{int } \{ \text{plan } y: \text{int } \{ z := x + y \} \} 3 4
\]
equivalent to \( z := 3 + 4 \). Here is a program to find the maximum value in nonempty list \( L \) in \( \log (\#L) \) time. (\( L \) is a variable, and its initial value is destroyed in the process.) We define \( \text{findmax } i j \) to find the maximum in the segment of \( L \) from index \( i \) to (but not including) index \( j \), reporting the result as \( L i \).

\[
\begin{align*}
\text{new findmax } \{ \text{plan } i: \Box L \text{plan } j: \Box L + 1 \}  \\
\text{[if } j - i \geq 2 \text{ ] [findmax } i \text{ (div } (i+j) 2) \parallel \text{findmax } (\text{div } (i+j) 2) j. \\
L := i \rightarrow L i \lor L \text{ (div } (i+j) 2) \mid L \]
\end{align*}
\]
After execution of \( \text{findmax } 0 (\#L) \), the maximum value in the initial list is \( L 0 \).

In the previous paragraphs, the parameter is a constant (note the single colon); it is not assignable. It is “by initial value”, so

\[
\text{plan } i: \text{int } \{ x := i, y := i \} (x+1)
\]
assigns both \( x \) and \( y \) to a value one greater than \( x \)’s initial value.

The second form of plan

\[
\text{plan } \text{simplename } :: \text{data } \{ \text{program} \}
\]
(note the double colon) creates a variable parameter. For example,

\[
\text{plan } x: \text{int } \{ x := 3 \}
\]
A plan with a variable parameter applies to a variable argument. But it cannot be applied to a variable appearing in the plan. This restriction is required for reasoning about the plan. This example plan can be applied to any variable, even one named \( x \), because that \( x \) is nonlocal, and is not the local variable \( x \) appearing in the plan. But the plan

\[
\text{plan } x: \text{int } \{ x := 3, y := 4 \}
\]
cannot be applied to variable \( y \).

Here is a plan named \( \text{norm} \) to reduce rational \( \text{num}/\text{denom} \) to lowest terms. If \( a=8 \) and \( b=12 \), then after \( \text{norm } a \ b \), we have \( a=2 \) and \( b=3 \).

\[
\begin{align*}
\text{new norm } \{ \text{plan } \text{num } :: \text{nat} + 1 \} \text{plan } \text{denom } :: \text{nat} + 1 \text{ `normalize num/denom} \\
\text{[new gcd } = \langle a: (\text{nat} + 1) \rightarrow (b: (\text{nat} + 1) \rightarrow \text{greatest common divisor of } a \text{ and } b \\
\text{a } = b \Rightarrow a \equiv a < b \Rightarrow \text{gcd } a \ (b-a) = \text{gcd } (a-b) \ b) \rangle, \\
\text{new g } := \text{gcd num denom. num } := \text{num}/g. \text{ denom } := \text{denom}/g \}
\end{align*}
\]
The main use for variable parameters is probably to affect many files in the same way; for example, a plan to sort files.

The next form of plan

\[
\text{plan } \text{simplename } ! \text{ data } \{ \text{program} \}
\]
creates a plan with an output channel parameter. For example.

\[
\text{plan } c ! \text{ text } \{ c! \text{ “abc”} \}
\]
This plan can be applied to any channel that receives text. A plan with a channel parameter cannot be applied to a channel appearing in the plan. This example plan can be applied to any output channel, even one named \( c \), because that \( c \) is nonlocal, and is not the local channel \( c \) appearing in the plan. But
The final form of plan

\texttt{plan simplename ? data \ll program \rr}

creates a plan with an input channel parameter. For example.
\texttt{plan c? text \ll c? 3*char. screen! c?? \rr}

This plan can be applied to any input channel that delivers text. But
\texttt{plan c? text \ll c? text. d? text \rr}

cannot be applied to channel \texttt{d}. The channel names \texttt{keys} and \texttt{screen} cannot be omitted when they are used as an argument for a channel parameter.

The following program \texttt{pps} has three channel parameters. On the first, \texttt{a}, it reads the coefficients of a rational power series; on the second, \texttt{b}, it reads the coefficients of another rational power series; on the last, \texttt{c}, it writes the coefficients of the product power series.

\begin{verbatim}
new pps \ll plan a? rat \ll plan b? rat \ll plan c? rat \rr
\ll a? rat \ll b? rat. c! a??x b?? .
  new a0:= a?? \ll new b0:= b?? \ll new d? rat! 0.
  pps a b d
\ll \ll a? rat \ll b? rat. c! a0x b??+a??x b0.
  loop \ll a? rat \ll b? rat \ll d? rat. c! a0x b??+d??+a??x b0. \rr \rr \rr
\end{verbatim}

A plan can be named; a plan can be argumented; and a plan can be the body of a plan. A plan cannot be composed sequentially or concurrently; a plan cannot be used in an \texttt{if}-program, nor in a \texttt{case}-program, nor in a \texttt{for}-program, nor in a \texttt{result}-expression. A plan with several parameters that has been partially argumented is still a plan; a plan that has been fully argumented is no longer a plan, and can be used wherever any program can be used. For examples,

\begin{verbatim}
plan y: real \ll x:= xy \rr 3. z:= 2 \ `this is allowed
plan y: real \ll x:= xy \rr. z:= 2 \ `this is not allowed
\end{verbatim}

\section*{Dictionary Definition}

Dictionaries are the way you organize your programs and data. Dictionaries exist only in the persistent scope, outside all local scopes. Dictionary definition has the form

\begin{verbatim}
new newname _
\end{verbatim}

When you define a \texttt{new} dictionary, the name must differ from all names in the persistent scope for which you are on one or both of its permit-lists (see \texttt{Permit}). The newname becomes a dictionary name. To create a new dictionary named \texttt{abc}, write

\begin{verbatim}
new abc _
\end{verbatim}

(It does not matter whether there are spaces between the name and the underscore.)

Now you can define names within this dictionary. A name being defined in a dictionary must not already be defined in that dictionary. Each name in a dictionary is defined, using the keyword \texttt{new} and a compound name, to be one of the following: a variable name, a constant name, a data name, a program name, a channel name, a unit name, or a dictionary name. For example,

\begin{verbatim}
new abc_x:= 2
\end{verbatim}

defines \texttt{x} in dictionary \texttt{abc} to be the constant \texttt{2}. (It does not matter whether there are spaces before or after the underscore.) This constant can then be used as \texttt{abc_x}. To define new dictionary \texttt{def} within dictionary \texttt{abc} write

\begin{verbatim}
new abc_def _
\end{verbatim}
When a name in a dictionary is defined to be a dictionary, this dictionary also can contain names, some of which can be defined as dictionaries, and so on. So a dictionary can be a tree structure. Suppose there is a dictionary named ProTem within which there is a dictionary named grammars within which there is a text named LL1. Its name is ProTem_grammars_LL1.

A name within a dictionary can become undefined by old. For example,

```plaintext
old abc_x
```

does the definition of variable x in dictionary abc. And

```plaintext
old abc
```

does the definition of dictionary abc. When a name becomes undefined, what it named remains in existence, anonymously, as long as something refers to it. When a dictionary becomes undefined, so do all the names within it.

Dictionary definitions, and definitions of names within dictionaries, do not obey the scope rules. If a dictionary definition occurs within scope brackets [], the dictionary is nonetheless in the persistent scope; its name must not already be defined in the persistent scope, but it can already be defined within the local scope. If a compound name definition occurs within scope brackets [], the definition is in a dictionary in the persistent scope; the name must not already be defined in that dictionary, but it can already be defined within the local scope. When a dictionary definition or definition of a name within a dictionary occurs within scope brackets, it does not become undefined at the end of the local scope. It does become undefined by using old, even within a local scope.

There is a dictionary named predefined (see Predefined Names).

**Synonym Definition**

Synonym definition has the form

```plaintext
new newname oldname
```

The newname becomes a synonym for the oldname. One use is to shorten all names that are deep within several dictionaries. For example, if dictionary a contains dictionary b, which contains dictionary c, which contains dictionary d, which contains variable x, then

```plaintext
new x a_b_c_d_x
```

shortens the name a_b_c_d_x to just x. The definition

```plaintext
new d a_b_c_d
```

shortens all names within a_b_c_d, for example, from a_b_c_d_x to d_x.

Another use is to rename something. To rename a to b, write

```plaintext
new b a. old a
```

**Format**

Although it is not part of the ProTem language, here are some suggested formatting (indentation) rules. The choice of alternative depends on the length of component data and programs.

```
A. B
or
A. for x:= A [B]
B
```

------------------------------------ ------------------------------------
More indentation would show the structure better, but it would crowd programs onto the right side of the page.

**Commands**

There are 11 commands in ProTem. They are not presented in the grammar, and they cannot be part of a stored program. They can be used only by a human at a keyboard. A command may be given at any time; it does not have to respect the grammatical structure of a program; it interrupts execution. Each command is the escape character combined with a letter. The commands are:

- `esc e`: enter or exit editor for saved program or data
- `esc a`: abort execution of program
- `esc s`: stop current session and start new session
- `esc p`: change dictionary permits
- `esc i`: change identity
- `esc u`: undo current session
- `esc n`: print names defined in current scope or persistent scope or in dictionary
- `esc m`: attach or modify or retrieve memo to defined name
- `esc d`: display source or object code for saved program or data
- `esc c`: generate context comments
- `esc v`: verify program according to a specification

**Edit**

The edit command `esc e` is used to modify an existing persistent program or data definition. It invokes a dialogue using keys and screen to determine which definition, and whether you have the proper permit. It then invokes an editor. In the editor, `esc e` exits the editor, and asks if you want to throw away the old definition, and save and compile the new definition. If the new definition has an error, you receive an error message, an error comment is inserted into the saved source, and the
compiled object code, when executed, prints “unable to execute [definition name]”. If you want to create a definition using the editor, first create the definition, for example, new p [ok], and then invoke the editor to modify it. If you want to delete a definition, use old.

Abort

It is essential to be able to abort the execution of a program, especially if you suspect that its execution will take forever. Use \texttt{esc a} to abort execution.

Session

Sessions are defined for security and error recovery. When the computer is turned on, a session begins. When some idle time passes (how much time is a parameter of the system and may be set to infinity), a session ends and a new one begins. When the computer is turned off, a session ends. The \texttt{esc s} command causes the current session to end and a new session to begin.

Sessions do not define the lifetime of definitions. A definition in the persistent scope, outside all [] pairs, lasts from the execution of the definition (\texttt{new}) to the execution of the corresponding name removal (\texttt{old}). This may be less than a session, or more than a session. Turning off the computer should not cut the power instantly, but should first cause the values of any variables in the persistent scope to be saved in nonvolatile memory.

At the start of each session, channels are reinitialized, the predefined text \texttt{session} is reinitialized to “”, and the programmer's identity is requested. If a programmer does not yet have an identity, they are invited to create one, and their persistent scope contains only the predefined dictionary (see Persistent Names). If they do have an identity, it is used to connect the programmer to their saved persistent scope. It is also used to determine a programmer's right to use or change definitions in the persistent scope (see Permit).

The predefined name \texttt{session} is a text consisting of all keystrokes since the start of the current session. (This is quite practical: an hour's hard work produces only 10kbytes of keystrokes.) This text can be saved as a record of work done, or for error recovery (see Undo).

Permit

Each definition in the persistent scope has two permit-lists of identities of people:
- use-permit-list: those who are permitted to use the definition
- change-permit-list: those who are permitted to change or edit the definition, delete (\texttt{old}) the definition, and change these two permit-lists attached to the definition

When you define a \texttt{new} name in the persistent scope, the name must differ from all names in the persistent scope for which you are on one or both of its permit-lists. (There may be several definitions of the same name with disjoint permit lists.) The two permit-lists of the newly defined name are just the name's creator. A permit-list is changed by means of the \texttt{esc p} command. The command starts a dialogue using keys and screen to ask which definition, to check if the person issuing the command is on the change-permit-list for that definition, to determine what change is wanted, and determine if the change would create overlapping permit-lists for the same name, and then, if all is well, to make the change. Definitions within a dictionary do not have permit-lists.

Permit-lists are not actually lists, but bunches of identities. The name everyone is predefined as the
bunch of all identities. Dictionary predefined has use-permit-list everyone and change-permit-list only its creator.

If you have just been sent an identity on channel id, you can save it under the name Josh by the constant definition

new Josh:= id??

You might start a changeable group of identities named condoBoard by the variable definition

new condoBoard: everyone:= Josh, Claire, Martin, Amanda

This is how you can maintain permit-lists to grant limited access to programs and data you create.

Identity

The esc i command is used to change identity, retaining one's persistent scope and permits. This command is used if one's identity is forgotten or compromised.

Undo

The command esc u undoes a session (except for inputs and outputs and session). Implementing it requires capturing the state at the start of a session. On many computers, returning to the prior state may be cheap; nonvolatile memory (that does not require power) contains the state as it was at the start of the current session, and volatile memory (that requires power) contains the current state.

After undo, you can capture the current value of session, let's call it recovery,

new recovery: text:= session

then reassign (or edit) recovery, and then execute the result by writing exec recovery. This gives us perfectly flexible error recovery for the modest cost of a keystroke file.

Names

The command esc n begins a dialogue using keys and screen to determine whether you want the names defined in the current scope, the persistent scope, or in a dictionary. In the persistent scope you will receive only the names for which you are on the use-permit-list. In a dictionary for which you are on the use-permit-list you will receive only the first-level names, not the names in subdictionaries.

Memo

Each definition can optionally have a memo attached to it. The memo might explain the purpose or use of the definition. It is there to be read by a human, not for execution. A memo is similar to a comment that you would make at the point of definition, but differs in that you can retrieve it anytime. The command esc m starts a dialogue using keys and screen to determine which name (simple or compound), whether you want to attach a new memo, modify an existing memo, or retrieve an existing memo, and checks whether you are on the appropriate permit-list. For example, you may say that you want to attach the memo

This variable accumulates the sum of the products.

to name x. Asking for the memo attached to predefined name e prints

\[ e:= 2.718281828459045 \text{ (approximately) constant}\] The base of the natural logarithms.
Display

The command $\text{esc d}$ starts a dialogue using $\text{keys}$ and $\text{screen}$ to determine the name (simple or compound) of the program or data whose source or object code you want to view, and checks whether you are on the use-permit-list.

Context

The command $\text{esc c}$ starts a dialogue using $\text{keys}$ and $\text{screen}$ to determine the program, bracketed by $[\ ]$, for which context comments are wanted. The comments are then generated. These comments say which nonlocal names are used, and in what way they are used. Here is the format.

```
`input: on these channels
`output: on these channels
`use: the values of these variables, constants, data names, units, and function names
`assign: these variables
`call: these program names and plan names
`refer: to these dictionaries
```

If there already are comments in this format, they are replaced. (For examples of context comments, see Example Programs.) Additionally, a programmer may want to include comments like

```
`spec: specification
`pre: precondition
`post: postcondition
`inv: invariant
```

but these are not generated by $\text{esc c}$.

Verify

The command $\text{esc v}$ starts a dialogue using $\text{keys}$ and $\text{screen}$ to determine the program, bracketed by $[\ ]$ and named by a fancy name, for which verification is wanted. The verification is then attempted. (See Program Definition and Named Program.)

Persistent Names

Outside all scope limiters, $[\ ]$ and $\langle \rangle$, is the persistent scope (see Scope). A name defined by $\text{new}$ in the persistent scope is called a persistent name. Its scope ends only with $\text{old}$. Its scope does not end with the end of a computing session (see Session), not even by turning off the computer, which should not cut the power instantly, but should first cause the values of any variables in the persistent scope to be saved in nonvolatile memory. Persistent variables serve as “files”.

The persistent scope can be shared among a community of programmers. Each definition in the persistent scope has two permit-lists to say who can use, change, or delete the definition (see Permit). The $\text{predefined}$ dictionary can be used by everyone, but changed only by its creator.

Sharing creates timing problems. The implementation must ensure that any change to a shared definition, including assigning a shared variable (changing a file), does not overlap with any other change to that shared name, nor with any use of that shared name.
Predefined Names

The predefined names are defined in dictionary predefined in the persistent scope (see Scope). Predefined names can be redefined. For example, one of the predefined names is the imaginary number \( i \) (a square root of \(-1\)). You may also want to define a local variable \( i \). If you do, you can still refer to the predefined \( i \) as predefined_\( i \) (unless you have also covered the name predefined with a local definition). If predefined name \( i \) is covered by a definition outside the local scope where you are working, you can get back the simple name \( i \) as the predefined imaginary number in these three ways:

\[
\begin{align*}
\text{new } i &:= \text{predefined}_i & \text{`constant definition} \\
\text{new } i \ \text{predefined}_i & & \text{`synonym definition} \\
\text{new } i &:= 0 \& 1 & \text{`constant definition}
\end{align*}
\]

Dictionary predefined has use-permit-list everyone and change-permit-list only its creator (see Permit). Unless these permit lists are changed by its creator, only its creator can add new definitions to the predefined dictionary. For example,

\[
\text{new predefined_amaze} = \langle n: \text{nat} \rightarrow n+2 \rangle
\]

Likewise, only the predefined dictionary's creator can remove definitions from the predefined dictionary. For example,

\[
\text{old predefined_amaze}
\]

The command \( \text{esc} \ n \) is used to list the names in the predefined dictionary. The command \( \text{esc} \ m \) is used to get a description of a predefined name.

Here are the predefined names. Some definitions use \( \$ \) (those), defined in a Practical Theory of Programming. Each name is one of:

- variable
  - at present, there are no predefined variables
- constant
  - evaluated; not assignable
- data
  - unevaluated; evaluation upon use; not assignable
- program
  - unexecuted; execution upon use
- channel
  - reinitialized at the start of each session
- unit
  - unrelated to other predefined units
- dictionary
  - at present, there are no predefined dictionaries

\[
\begin{align*}
\text{abs: com} & \rightarrow \text{real data} \quad \text{Absolute value.} \quad \text{abs } x = \sqrt{\text{re } x^2 + \text{im } x^2}. \\
\text{all} & = \text{com, char, bin, fall, } [\text{*all}] \quad \text{data} \quad \text{All ProTem items.} \\
\text{arc: com} & \rightarrow \$<r: \text{real} \rightarrow 0 \leq r < 2\pi) \quad \text{data} \quad \text{The angle or arc of a complex number.} \\
\text{arccos: } & \$<r: \text{real} \rightarrow -1 \leq r \leq +1) \rightarrow \$<r: \text{real} \rightarrow 0 < r < \pi/2) \quad \text{data} \quad \text{A trigonometric function.} \\
\text{arcsin: } & \$<r: \text{real} \rightarrow -1 \leq r \leq +1) \rightarrow \$<r: \text{real} \rightarrow 0 < r < \pi/2) \quad \text{data} \quad \text{A trigonometric function.} \\
\text{arctan: real} & \rightarrow \$<r: \text{real} \rightarrow 0 < r < \pi/2) \quad \text{data} \quad \text{A trigonometric function.} \\
\text{await} & \quad \text{program} \quad \text{A plan with one constant parameter of type realss.} \quad \text{If the argument represents the present or a future time, its execution does nothing but takes time until the instant given by the argument.} \quad \text{If the argument represents the present or a past time, its execution does nothing and takes no time. See time and wait and s.} \\
\text{back: } & \text{*nat} \rightarrow \text{*nat data} \quad \text{If } i \text{ is an item, } \text{back } (s; i) = s. \\
\text{bin: } & \quad \text{T, } \bot \quad \text{constant} \quad \text{The binary values.} \\
\text{bold: text} & \rightarrow \text{text data} \quad \text{Same text but in bold font.} \\
\text{ceil: real} & \rightarrow \text{int data} \quad r \leq \text{ceil } r < r+1 \\
\text{char} & \quad \text{data} \quad \text{The characters.} \\
\text{charnat: char} & \rightarrow \text{nat data} \quad \text{A one-to-one function with inverse natchar.} \quad \text{The encoding might be}\
\end{align*}
\]
extended ASCII or unicode. Character combinations, for example shift-option-a, also have numeric encodings.

click: char constant The click character.
com = real&real data The complex numbers.
cos: real → $(r: real → -1 ≤ r ≤ +1) data A trigonometric function.
cosh: com→com data A hyperbolic function.
cursor: nat; nat data A data name whose value is the current cursor position.
delete: char constant The delete or backspace character.
digit: char constant The decimal digits.
div: real → $(r: real → r>0) → int data div a d is the integer quotient when a is divided by d.
  \(0 \leq \text{mod } a \ d < d\) \(\land (a = \text{div } a \ d \times d + \text{mod } a \ d)\)
doubleclick: char constant The doubleclick character.
e := 2.718281828459045 (approximately) constant The base of the natural logarithms.
end: char constant The end-of-file character. It is greater than all other characters.
even: int→bin data A function that says whether its argument is even.
everyone data All identities.
exec program A plan with one text parameter. If the argument represents a ProTem program, the execution is that of the represented program. It “unquotes” its argument. If applied to “x := x + 1”, the “x” refers to whatever x refers to at the location where exec “x := x + 1” occurs.
  If the argument does not represent a ProTem program, execution displays an error message.
exp: com→com data The exponential function. \(exp \ x = e^x\).
false := \bot constant A binary value.
find: all→*all→nat data If i is an item in string S, then \(\text{find } i S\) is the index of its first occurrence; if not, then \(\text{find } i S = \leftrightarrow S\).
fit: int→text→text data If \(i \geq 0\) then \(\text{fit } i t\) is a text of length i obtained from t by either chopping off excess characters from the right end or by extending t with spaces on the right end. If \(i \leq 0\) then \(\text{fit } i t\) is a text of length \(-i\) obtained from t by either chopping off excess characters from the left end or by extending t with spaces on the left end.
floor: real→int data floor r \leq r < 1 + floor r
form: nat→nat→(nat+1)→real→text data Format a real number. \(\text{form } d e w r\) is a text representing real r with the final digit rounded. d is the number of digits after the decimal point; if \(d=0\) the point is omitted. e is the number of digits in the exponent; if \(e>0\) the decimal point will be placed after the first significant digit; if \(e=0\) the \(^\wedge\wedge\) is omitted and the decimal point will be placed as necessary. w is the total width; if \(w>0\) greater than necessary, leading blanks are added; if \(w<0\) less than sufficient, the text contains blobs.
  \(\text{form } 4 \ 1 \ 0 \ pi = " 3.1416 ^ \wedge \wedge 0"\) \(\text{form } 2 \ 0 \ 6 (-pi) = "-3.14"\)
  \(\text{form } 0 \ 0 \ 3 \ 5 = " \ 5"\) \(\text{form } 0 \ 0 \ 3 \ (-5) = "-5"\) \(\text{form } 0 \ 0 \ 2 \ 123 = "•••"\).
g unit Representing mass in grams.
i := 0&1 constant An imaginary number: a square root of \(-1\).
im: com→real data The imaginary part of a complex number.
infinity := \infty constant An infinite number, greater than all other numbers.
int = nat; nat data The integers.
italic: text→text data Same text but in italic font.
keys? text! “ ” channel To the program that monitors key presses, it is an output channel; to all other programs, it is an input channel.
lb: $(r: real → r>0) → real data The binary (base 2) logarithm.
ln: $(r: real → r>0) → real data The natural or Napierian (base e) logarithm.
log: $(r: real → r>0) → real data The common (base 10) logarithm.
m unit Representing distance in meters.
m matches: *all→*all→nat data If pattern occurs within subject, then match pattern subject is the
index of its first occurrence. If not, then \texttt{match pattern subject = ↔ subject}.

\texttt{maxint: int constant} The maximum representable integer (machine dependent).

\texttt{maxnat: nat constant} The maximum representable natural (machine dependent).

\texttt{microphone? *sound! silence channel} To the microphone, it is an output channel; to all other programs, it is an input channel.

\texttt{minint: int constant} The minimum representable integer (machine dependent).

\texttt{mod: real \to \{r: real \to r>0\} \to real data} \texttt{mod a d} is the remainder when \texttt{a} is divided by \texttt{d}.

\[ (0 \leq \text{mod a d} < \text{d}) \land (a = \text{div a d} \times \text{d} + \text{mod a d}) \]

\texttt{movie = *picture data} A string of pictures.

\texttt{nat = 0,..∞ data} The natural numbers.

\texttt{natchar: nat\to char data} A one-to-one function with inverse \texttt{charnat}. The encoding might be extended ASCII or unicode. Character combinations, for example shift-option-a, also have numeric encodings.

\texttt{nil constant} The empty string.

\texttt{nl: char constant} The new line character or next line character or return character.

\texttt{null constant} The empty bunch.

\texttt{numpat: text constant} A text pattern for numbers, useful for reading a number from a text channel.

\texttt{odd: int\to bin data} A function that says whether its argument is odd.

\texttt{ok program} A program whose execution does nothing and takes no time.

\texttt{ord = real, char, bin, *all, *ord, [ord] data} The ordered type, for which \texttt{< > ≤ ≥} are defined.

\texttt{pi:= 3.141592653589793 (approximately) constant} The ratio of a circle’s circumference to its diameter.

\texttt{picture = [x*y*[0,..z]] data} where \texttt{x} is the number of pixels in the horizontal dimension, \texttt{y} is the number in the vertical dimension, and \texttt{z} is the number of pixel values.

\texttt{point data} A function that applies to a list and gives its deep domain (a bunch of strings of indexes).

It is a signal to the implementation that the strings in it will be used only as indexes to the list. It can therefore be implemented as a memory address (pointer).

\texttt{pre: char\to char constant} The character predecessor function.

\texttt{printer? text! "" channel} To the printer, it is an input channel; to all other programs, it is an output channel.

\texttt{quote: *all\to text data} produces a text representation of its argument. The argument is evaluated before quoting. See \texttt{unquote} and \texttt{form}.

\texttt{randNat: nat\to nat\to nat data} A reasonably uniform function, dependent on a hidden variable, over the interval from (including) the first argument to (excluding) the second argument.

\texttt{randNatInit program} A plan with one constant natural parameter. Its execution assigns a hidden variable to the natural value.

\texttt{randNatNext program} Its execution assigns a hidden variable to the next value in a random sequence.

\texttt{randReal: real\to real\to real data} A reasonably uniform function, dependent on a hidden variable, over the interval between the arguments.

\texttt{randRealInit program} A plan with one constant real parameter. Its execution assigns a hidden variable to the real value.

\texttt{randRealNext program} Its execution assigns a hidden variable to the next value in a random sequence.

\texttt{rat = int/(nat+1) data} The rational numbers.

\texttt{re: com\to real data} The real part of a complex number.

\texttt{real data} The real numbers.

\texttt{round: real\to int data} \texttt{r\to0.5 \leq round r < r+0.5}

\texttt{s unit} Representing time in seconds.

\texttt{screen? text! "" channel} To the screen, it is an input channel; to all other programs, it is an output channel.
session: text data All keystrokes on channel keys since the start of a session.
sign: real → (−1, 0, 1) data The sign of a real number.
silence: sound data The silent sound.
sin: real → ¢(r: real → −1 ≤ r ≤ +1) data A trigonometric function.
sinh: com→com data A hyperbolic function.
sort: *ord→*ord data Sorts in nondecreasing order.
sound data The sounds.
speaker? *sound! silence channel To the speaker, it is an input channel; to all other programs, it is an output channel.
sqrt: com→com data The principal square root.
stop program Its execution does nothing and takes forever so that no computation can follow.
sub: *all→nat→nat→*all→*all data sub s n m t is a string formed from string s by replacing the substring from index n to index m with string t. The substring being replaced s(n;..m) does not have to be the same length as the string t replacing it. If n=m this is insertion. If t=nil this is deletion. sub s n m t = s(0;..n); t; s(m;..⇒s)
subst: *all→all→all→*all data subst s x y is a string formed from string s by replacing all occurrences of item x with item y.
suc: char→char constant The character successor function.
tab: char constant The tab character.
tan: real→real data A trigonometric function.
tanh: com→com data A hyperbolic function.
text = *char data
textid: text→everyone data A not-invertible injective function.
texttime: text→(realxs, “error”) data If the argument represents a time, possibly preceded by space, tab, and new line characters, possibly followed by space, tab, and new line characters, the result is the represented time in seconds since 2000 January 1 at 0:00 UTC (the midnight that begins 2000 January 1 at longitude 0). Times before then are negative. For example, texttime “1947 September 16 at 14:24:32.5 UTC–5” = −68675727.5×s. Otherwise the result is “error”.
thisdata: text data A text representing the name of the smallest enclosing data definition, or the empty text “” if there is no enclosing data definition. For example,
  new abc = thisdata
defines abc to be the text “abc”.
thisprogram: text data A text representing the name of the smallest enclosing program definition, or the empty text “” if there is no enclosing program definition. For example,
  new abc [ !thisprogram ]
defines abc to be a program whose execution sends the text “abc” to the screen.
time? realxs! 0xs channel To the time provider, it is an output channel. To all other programs, it is an input channel that gives the current time in seconds since 2000 January 1 at 0:00 UTC (the midnight that begins 2000 January 1 at longitude 0). It is a monitor, which is a shared variable using channel syntax; values on channel time are not buffered.
timetext: (realxs)→rat→text data Given the time in seconds since 2000 January 1 at 0:00 UTC (the midnight that begins 2000 January 1 at longitude 0), and a time zone, the result is a readable text. Times before then are negative. For example,
  timetext (−68675727.5×s) (−5) = “1947 September 16 at 14:24:32.5 UTC–5”
trim: text→text data A text formed from the argument by removing all leading and trailing space, tab, and new line characters.
true:= ⊤ constant A binary value.
unquote: text→*all data If the argument represents a ProTem data expression, the result is the value
of the represented data. The argument is evaluated after unquoting. If the argument does not
represent a ProTem data expression, the result is “error”. See *quote*.

**wait** program A plan with one constant parameter of type *reals* . If the argument is nonnegative,
its execution does nothing and takes the time given by the argument. If the argument is
nonpositive, its execution does nothing and takes no time. See *await* and *time* and *s* .

**Miscellaneous**

As a character within a text, the left- and right-double-quote characters must be underlined. For
example, “Just say “no”.”. As a character within a text, an underlined left- and right-double-quote
character must be underlined again. And so on. Thus every character can occur within a text. But
we cannot write a self-reproducing expression with this convention. For that purpose, we need
another convention, such as repeating the left- and right-double-quote characters within a text. For
example, “Just say ““no””.”. Using this convention, here is a self-reproducing expression (perform
the indexing to see what you get).

```
""""(0;0;(0;..32);31;31;(1;..31))""""(0;0;(0;..32);31;31;(1;..31))
```

Sounds and pictures are data structures. This part of ProTem is not yet designed. Perhaps a picture
is an element of \([x\times y\times(0..z)]\) where \(x\) is the number of pixels in the horizontal direction, \(y\) is the
number of pixels in the vertical direction, and \(z\) is the number of pixel values. A picture could
therefore be expressed in the same way as any other two-dimensional array, and one could refer to
the pixel in column 3 and row 4 of picture \(p\) as \(p\[3\,4\]\). Perhaps a movie is a string of pictures. The
operations on movies would be those of strings, such as substring and join. To help in the creation of
movies, one of the pixel values should be transparent, and one of the operations on pictures should
be overlaying one picture on another. Predefined *silence* is a sound, and predefined *sound* is all
sounds. Sounds are input on channel *microphone* ; pictures are input on channel *camera* . A
constant can be defined as a sound or picture. A variable can be assigned to a sound or picture.
Sounds and pictures can be included in a data structure, and manipulated using the operators on that
data structure. Sounds can be output on channel *speaker* ; pictures can be output on some not-yet-
determined channel.

The ProTem equivalent of enumerated type is shown here.

```protem
new color: = “red”, “green”, “blue”.
new brush: color: = “red”
```

The ProTem equivalent of the record type (structure type) is as follows.

```protem
new person: = “name” \rightarrow text | “age” \rightarrow nat.
new p: person: = “name” \rightarrow “Josh” | “age” \rightarrow 16
```

The fields of \(p\) can be selected by an argument, for example

```
! p “name”
```

prints the text “Josh”. The value of \(p\) can be changed using a function arrow and selective union.

```protem
p: = “age” \rightarrow 17 | p.
p: = “name” \rightarrow “Amanda” | “age” \rightarrow 2
```

We can even have a whole file (string) of records

```protem
new file: *person: = nil
```

and join new records onto its end.

```
file: = file; p
```

The efficiency of pointers is obtained through the use of the predefined function *point*. When
applied to a list argument, it yields the deep domain of the list. For example,
point [10; [11; 12]; 13] = 0 , 1; (0, 1) , 2 = 0 , 1; 0 , 1; 1 , 2
The use of point is a signal to the implementation that its strings of natural numbers will be used only as indexes into the list (and the implementation will check that this is so). For example, we can define a linked list G as follows.

\[
\text{new } G: \{\text{"name" } \rightarrow \text{ text } | \text{ "next" } \rightarrow \text{ point } G\} := \{\text{"name" } \rightarrow \text{ end } | \text{ "next" } \rightarrow 0\}.
\]

We can add a constant to first or subtract a constant from it, for example

\[
\text{first} := \text{first} + 1
\]
and similarly for the “next” field of each record of G. But we can ultimately use them only as indexes into G, for example

\[
G := \text{first} \rightarrow (\text{"name" } \rightarrow \text{ “Aaron” } | \text{ "next" } \rightarrow \text{ first}) | G
\]
With this limited use, the implementation of these indexes can be memory addresses. This way we obtain all the performance benefits of pointers without destroying the logic of our language.

The previous example, with linked list G, does not show the full generality of point. Here is a tree-structured example.

\[
\text{new } tree = [\text{nil}], [\text{tree}; \text{all}; \text{tree}].
\]
\[
\text{new } t: \text{tree} := [\text{nil}].
\]
\[
\text{new } p: \text{point } t := \text{nil}
\]
To move p down to the left in the tree we reassign it this way:

\[
p := p; 0
\]
To move it down to the right, reassign it this way:

\[
p := p; 2
\]
Thus p is a string of indexes indicating a subtree t@p of t. We can replace this subtree with tree s using the assignment

\[
t := p \rightarrow s | t
\]
We can express the information at the node indicated by p as

\[
t @ p \ 1 \ or \ t @ (p; 1)
\]
and we can replace the information at this node with the integer 6 using the assignment

\[
t := (p; 1) \rightarrow 6 | t
\]
To move up in the tree, we just remove the final item of p, and to make that easy, the predefined

\[
\text{new } \text{back} = \langle \text{p: (*nat)} \rightarrow \text{p} (0; .. \leftrightarrow \text{p} – 1)\rangle
\]
allows us to move p up to its parent by writing

\[
p := \text{back } p
\]
The “procedure”, “method”, or “function” of some other programming languages is a combination of naming, scope, and parameter(s). For example,

\[
\text{new } \text{transform} \ [\text{plan magnification: real} \ [\text{plan translation: real} \ [x := \text{magnification}\times x + \text{translation}]]]
\]
Here is a definition of a plan with one parameter

\[
\text{new } \text{translate} \ [\text{transform } 1]
\]
formed by providing one argument to a two-parameter plan. To provide an argument for just the second parameter is a little more awkward, but not too bad.

\[
\text{new } \text{magnify} \ [\text{plan magnification: real} \ [\text{transform magnification } 0]]
\]
We can now obtain a three-times magnification of x in either of these ways.

\[
magnify 3
\]
\[
\text{transform } 3 \ 0
\]
In some other programming languages, the “function” is a combination of naming, scope,
parameter(s), and result-expression. For example,

\[
\text{new factorial} = \langle n: \text{nat} \to \text{result} f: \text{nat}:= 1 \ [\text{for } i:= 1;..n+1 \ [f:= f\times i]]\rangle
\]

Exception handling is provided by | or if or ⊨ ⫤. For example,

\[
\text{new divide} = \langle \text{dividend: com} \to \langle \text{divisor: com} \to
\text{divisor = 0} \iff \text{“zero divide”} ⊨ \text{dividend} / \text{divisor}\rangle\rangle
\]

Then

\[
\text{divide: com} \to \text{com} \to (\text{com, “zero divide”})
\]

The selective union operator applies its left side to an argument if that argument is in the stated domain of its left side; otherwise it applies its right side. Let us define

\[
\text{new weekday} = \langle d: (0,..7) \to 1 \leq d \leq 5 \rangle
\]

Then in the expression

\[
(\text{weekday} \mid \text{all} \to \text{“domain error”} ) i
\]

if \( i \) fails to be an integer in the range 0,..7 , the left side of | “catches” the exception and “throws” it to the right side, where it is “handled”.

Input choice, as in CSP, can be obtained as follows.

\[
\text{inputchoice} [\text{if } c!! [c? \text{numpat}; nl. P] \text{ else [if } d!! [d? \text{numpat}; nl. Q] \text{ else [inputchoice]]}]
\]

In the persistent scope, ProTem functions as an operating system, where programs are executed as soon as they are entered. Unix directories are dictionaries. Unix files are variables. Unix rm is ProTem's old. Unix mv is a synonym definition followed by old. The commands esc n and esc m are the Unix ls and man commands. ProTem's esc p replaces Unix's chmod. The effect of Unix pipes is obtained by channel parameters. For example, suppose trim is a plan to trim off leading and following blanks and tabs from lines of text, and sort is a plan to sort texts. (Please excuse the informal body since it's not the point.)

\[
\text{new trim [plan in? text [plan out! text [Repeatedly read from in , trim off leading and trailing space, output to out , until end is read and output.]]]]}
\]

\[
\text{new sort [plan in? text [plan out! text [Repeatedly read from in until end is read, and output the sorted texts and end to out.]]]]}
\]

We can feed the output from trim to the input of sort by defining a channel for the purpose. If the original input comes from keys , and the final output goes to screen , then

\[
\text{new pipe? text!”“. trim keys pipe. sort pipe screen. old pipe}
\]

Even better:

\[
\text{new pipe? text!”“. trim keys pipe || sort pipe screen. old pipe}
\]

If sort wants input before it is available from trim , sort waits. If trim sends output before sort wants it, it is buffered.

An implementation may provide plans for a variety of languages. For example, it may provide a plan named Python , with one text parameter, whose execution is that of the Python fragment represented by the argument. It may provide asm , whose execution is that of the assembly-language program represented by the argument.

ProTem considers object orientation to be a programming style, rather than a programming-language style, or collection of language features. Object-oriented programming (as a style of programming) can be done in ProTem. Data structures, and the functions and procedures that access and update them, can be defined together in one dictionary. If many objects of the same type are wanted, the type can be parameterized as appropriate, defined once, and used many times.

To execute a program stored on someone else's computer, just invoke that remote program using its
full address (computername_programname). For efficiency, it might be best to compile that remote
program for your own computer and run it locally. Any nonlocal names (variables, channels, and so
on) refer to entities on the computer where the program is compiled.

The keyword else is redundant; it could be deleted everywhere it occurs, reducing the number of
keywords to 7. But I think it is useful redundancy, for both error detection and readability.

**Intentionally Omitted Features**

Each of the following omitted features is a syntactic convenience, and it's no trouble to add to the
language. But they make the language larger, and that's a cost. And they move away from the form
needed for verification. So they are not included in ProTem.

- **assertion**
  - `assert x$\leq y` means `if \(\neg(x \leq y)\)⟦! “assert failure”. stop⟧`
- **item assignment**
  - `S\3:= 5` means `S:= S<3>5`
  - `L 3:= 5` means `L:= 3\rightarrow 5 \mid L`
  - `L 3 4:= 5` means `L:= (3;4)\rightarrow 5 \mid L`
- **name grouping**
  - `new x, y: int:= 0` means `new x: int:= 0 \parallel new y: int:= 0`
  - `old x, y means `old x \parallel old y`
  - `(a, b: nat \rightarrow a+b)` means `⟦a: nat \rightarrow (b: nat \rightarrow a+b)⟧`
- **plan**
  - `plan a, b: nat [x:= a+b]` means `plan a: nat [plan b: nat [x:= a+b]]`
  - `x, y:= 0` means `x:= 0 \parallel y:= 0`
- **looping constructs**
  - `while n>0 [n:= n–1] until n=0` means `while [if n>0 [n:= n–1. while]]`
  - `repeat [n:= n–1] until n=0` means `repeat [n:= n–1. if \(\neg(n=0)\) [repeat]]`
  - `loop [n:= n–1. if n=0 [exit]. m:= m+1]` means `loop [n:= n–1. \if (n=0) [m:= m+1. loop]]`
- **return from program**
  - `if n=0 [return]. restOfProgram` means `if \(\neg(n=0)\) [restOfProgram]`

The assignment `L:= 3\rightarrow 5 \mid L` should be compiled the same as `L 3:= 5` would be if it were included
in ProTem; the list `L` should not be copied. The same for string item assignment. In the loop
`while [if n>0 [n:= n–1. while]]`

the last-action (tail recursive) call should be compiled as a branch (jump) instruction, with no stack
activity, the same as a while-loop would be if it were included. Omitting string and list item
assignment and special looping constructs should not cost execution time.

Dictionary and program parameters and arguments were considered and rejected.

- `plan simplename _ [ program ]` plan, parameter is dictionary
- `program dictionaryname` plan, dictionary argument
- `plan simplename [ program ]` plan, parameter is program
- `program program` plan, program argument

As a direct counterpart to the Unix cd command, we considered

- `open dictionaryname`
- `close dictionaryname`

to allow names in that dictionary to be referred to without stating the dictionary. For example, if we
have dictionary `abc`, and within it names `x` and `y`, we can refer to these names as `abc_x` and
`abc_y`. By saying
open abc
we can then refer to them as just $x$ and $y$. But the interaction between open and scope is complex, we can already refer to names within predefined, and we can shorten names by synonym definition, so we left out open and close.

There is no frame construct in ProTem, but [esc c] serves the same purpose.

In some languages there is a module or object construct for the purpose of grouping together related definitions. In ProTem, dictionaries serve that purpose.

Do we ever want the correcting pattern without an echo? For a password, we usually want to output some character, like •, for each key press, and program the corrections, rather than using the correcting pattern (see Read Password). If there is a reason anyone wants the correcting pattern without an echo, we can add it, but it will complicate the grammar.

Two binary operators $\Delta$ (nand) and $\nabla$ (nor) are missing. They aren't wanted very often, there aren't good keyboard substitutes for them, and they are easily synthesized:

$x\Delta y = \neg(x \land y)$
$x\nabla y = \neg(x \lor y)$.

**Implementation Philosophy**

Ideally, an implementation checks whether the text presented to it represents a program, and issues an error message if it does not. That check should include determining whether every variable assignment is to a value that is included in the type of the variable. That determination is most helpful if it can be made before execution; but if not, it is still helpful if it can be made during an execution attempt.

While not an error, there are also expressions that cannot or should not be evaluated further. That presents an implementation problem, but not a semantic problem. For example,

! −3 prints −3

ProTem does not evaluate the application of the negation operator − to the operand 3 (see Number Representation); it just prints the operator and operand. Similarly

! 1/0 should print 1/0

! [0; 1] 2 should print [0; 1] 2

! \langle r: rat → 5 \rangle (1/0) should print 5

! 1/0 = 1/0 should print \top

! [0; 1] 2 = [0; 1] 2 should print \top

Due to the difficulty of implementation, it is permissible for an implementation to behave differently.

No programming language has ever been, or will ever be, implemented entirely. Every programming language is infinite; every implementation is finite. There is always a program too big for the implementation. There is a multitude of size limitations: the parse stack might overflow, the dictionary (symbol table) might be too small, the forward branch fixup list might be exceeded, and so on. It would be ugly to define a programming language by listing all the size limitations of programs. And it would be counter-productive because it would exclude implementations that can accommodate larger programs.

Whenever a program exceeds a size limitation, the implementation should not say “Error: limitation exceeded.”, because the program is not in error. The implementation should say “Apology: this
implementation is too limited to accommodate your program.”. An “error” message tells a programmer to correct the error; there is no other option. An “apology” message gives the programmer 3 options: change the program to live within the limitation; change the implementation options to increase the limit that was exceeded; take the program to a different implementation.

Natural numbers and integers are usually limited to those that are representable in a specific number of bits, for example, 32 bits. This is a size limitation, just the same as other size limitations. It is more complicated and uglier to define arithmetic within finite limitations than to define the naturals and the integers. And it is counter-productive to do so, because it excludes an implementation with 64-bit arithmetic. As with other implementation limitations, numeric overflow should not get an “error” message; it should get an “apology” message.

Floating-point numbers and arithmetic should never be offered as a language feature. The programmer wants rational or real numbers and arithmetic, but may be willing to accept the floating-point approximation for the sake of efficiency. Floating-point, with a specific number of bits, is an implementation limitation. Any alternative to floating-point that increases the accuracy without taking too much time or space should be welcome.

ProTem is a rich programming system, offering many kinds of data and operators on data, and many ways to structure a computation. Some features may be difficult to implement. And some features may be of little use to most programmers. It may be a wise decision not to implement some features. For example, an implementer might decide that in a variable definition, the type must be one of

\[
  \text{nat int rat bin text } [n\text{type}]
\]

where \(n\) is a natural number and \(\text{type}\) is any of these types just listed. An implementer may decide not to implement concurrent execution. No-one can complain that the complete language is not implemented, since it is impossible to completely implement any language. But ProTem is defined to allow all type expressions that make sense, and to allow concurrency, so the next implementation can accommodate programs that previous implementations could not accommodate.

**Example Programs**

**Portation Simulation**

```
new simport ` a program to simulate portation
][
  `input: keys time
  `output: screen
  `use: ceil m nat nil nl point quote rat real s sqrt unquote
  `call: await stop

  ` Distance between control boxes is always 1 m.
  ` Merges do not overlap, so there's at most 1 corresponding box on the merging portway.
  ` Each divergence has a left branch and a right branch: there's no straight.
  ` Leading to a divergence, boxes record only one square speed.

  ` start of definitions

new km:= 1000×m. new h:= 60×60×s. ` kilometer and hour

new maxaccel:= 1.5×m/s/s. ` maximum deceleration = –maxaccel
new speedlimit:= 60×km/h. ` speed limit is 60 km/h everywhere
```
new cushion := 1 × s. ` reaction time for all porters
new impatience := 10/s. ` acceleration factor
new maxdistance := ceil(speedlimit^2 / (2 × maxaccel)). ` max search distance ahead
new numporters := 120.
new numboxes := 7480.
new visualDelayTime := 0.5 × s. ` for human viewing

new porter. ` so porter can be referenced before it is defined

new box: [numboxes * ((“ahead left”, “ahead right”, “behind left”, “behind right”) → point box
| “beside” → point box
| “above” → point porter, numporters
| (“horizontal”, “vertical”) → nat)] ` box position on screen
:= [numboxes * ((“ahead left”, “ahead right”, “behind left”, “behind right”) → 0
| “beside” → 0
| “above” → numporters ` indicates no porter above
| (“horizontal”, “vertical”) → 0 )].

new Porter: [numporters * ( “below” → point box ` what’s beneath
| “arrival time” → real × s ` arrival time at this box
| “speed” → real × m/s )] ` current speed
:= [numporters * ( “below” → 0
| “arrival time” → 0 × s
| “speed” → 0 × m/s )].

new draw [plan b: nat [plan c: “grey”, “blue”, “red” [UNFINISHED]]]. `end of draw
` draws a box at screen position (box b “horizontal”) (box b “vertical”) of color c.
` “grey” means no porter present, “blue” means porter present, “red” means crash
` UNFINISHED because graphical output has not yet been designed

` end of definitions, start of initialization

for b := 0;..numboxes
[ ! “What box is ahead-left of box ”; b; “? ”. ?!
box := (b; “ahead left”) → ?? | (?; “behind left”) → b \| box.
! “What box is ahead-right of box ”; b; “? ”. ?!
box := (b; “ahead right”) → ?? | (?; “behind right”) → b \| box.
! “What box is beside box ”; b; “? ”. ?!
box := (b; “beside”) → ?? \| box.
! “What are the horizontal and vertical coordinates of box ”; b; “? ”. ?!
box := (b; “horizontal”) → ?? \| box.
?!. box := (b; “vertical”) → ?? \| box.
draw b “grey”]. ` default color; may be changed below

for p := 0;..numporters
[ ! “Porter ”; p; “ is over what box? ”. ?!
porter := (p; “below”) → ?? | porter. box := (?; “above”) → p \| box.
draw (??) “blue”].

randNatInit 123456789. ` initialize a random number generator
end of initialization, start of simulation

infiniteLoop

` time? reals. ` the time of the start of each iteration of the infiniteLoop

new p: point porter:= 0. ` the porter that arrived at its current position first
new t: reals:= ∞. ` t is a time, initially an infinite time
for q:= 0;..numporters [if porter q “arrival time” < t [t:= porter q “arrival time”, p:= q]].
old t.

new b:= porter p “below”. ` the box below porter p
new bb:= box b “beside”. ` the box beside b; if none then bb=b
new boxesToDo: *[point box; nat×m]:= nil.
   ` queue of boxes to be explored; their distances ahead of porter p
   ` queue is sorted by increasing distance ahead
   ` difference between any two distances in the queue is at most 1

` initialize boxesToDo
if bb = b [boxesToDo:= nil]
else [if box bb “above” = numporters [boxesToDo:= nil]]
   else [if porter (box bb “above”) “speed” < porter p “speed” [boxesToDo:= nil]]
   else [boxesToDo:= [bb; 0×m]]].
boxesToDo:= boxesToDo; [box b “ahead left”; 1×m].
if box b “ahead left” ≠ box b “ahead right” [boxesToDo:= boxesToDo; [box b “ahead right”; 1×m]].
old b. old bb.

new accel: reals×m/s/s:= maxaccel. ` acceleration for porter p

` using boxesToDo calculate accel for porter p

nextBox [new b:= (boxesToDo\0) 0. ` the box we are looking at
new d:= (boxesToDo\0) 1. ` its distance ahead of porter p
boxesToDo:= boxesToDo\1;..↔boxesToDo).]
if d≤maxdistance
   [new desiredspeed = ` according to porter pa
      ⟨pa: (point porter, numporters) →
      pa=numporters ⊨ speedlimit
      ⇒ ( sqrt ( porter pa “speed”^2 + 2×maxaccel×d
      + (maxaccel×cushion)^2 )
      − maxaccel×cushion) ∧ speedlimit).]
accel:=
   ( ( desiredspeed (box b “above”)
     ∧ desiredspeed (porter (box b “beside”) “above”))
   − porter p “speed”)
   × impatience)
   ∨ −maxaccel ∧ maxaccel.
if box b “above” = numporters = porter (box b “beside”) “above”
   [ ` add boxes ahead to queue and continue
     boxesToDo:= boxesToDo; [box b “ahead left”; d+1×m].
   if box b “ahead left” ≠ box b “ahead right”
\[ \text{boxesToDo} := \text{boxesToDo}; \{ \text{box \textit{b} “ahead right”; d+1} \times \text{m} \} \].
\text{nextBox} \\
\text{else} \text{[if} \leftrightarrow \text{boxesToDo} > 0 \text{[nextBox]]].
\text{old} \text{boxesToDo}.

` using \text{accel}, move \text{porter p} ahead one box
\text{new} \text{b: point box} := \text{porter p} “below”.
\text{box} := (\text{b; “porter”}) \rightarrow \text{numporters \mid box. draw b “grey”}. 
\text{randNatNext} \\
\text{b} := \text{box b (randNat 0 2 = 0) “ahead left” = “ahead right”). 
\text{if} \text{box b “porter” < numporters [] draw b “red”. stop} \text{.} \text{\_ crash}
\text{porter} := (p; “below”) \rightarrow b \mid \text{porter. box} := (\text{b; “above”) \rightarrow p \mid box. draw b “blue”}. 
\text{old} \text{b}.
\text{new} \text{speed} := \sqrt{\text{repair p “speed”}^2 + 2 \times \text{accel} \times \text{m}} \land \text{speedlimit}. 
\text{porter} := (p; “arrival time”) \rightarrow \text{porter p “arrival time”} + 2 \times \text{m}/(\text{porter p “speed”} + \text{speed}) \\
\mid (p; “speed”) \rightarrow \text{speed} \\
\mid \text{porter}.
\text{old} \text{speed. old} \text{accel. old} \text{p. \_ these olds aren’t really necessary}

\text{await} (\text{time}??+\text{visualDelayTime}). 
\text{infiniteLoop } ]]] \text{`end of simport}

\textbf{Quote Notation Lengths}

` program to compare quote notation lengths with numerator/denominator lengths

`\text{output: screen}
`\text{use: bin div even nat odd quote}

\text{new} \text{shl} = \langle n: \text{nat} \rightarrow \langle m: \text{nat} \rightarrow \text{\_ shift n left m places; n} \times 2^m \rangle \text{result} \text{r: \text{nat}:=} n [\text{for i:= 0;} \text{..m [r:= r} \times 2]]\rangle\rangle.

\text{new} \text{shr} = \langle n: \text{nat} \rightarrow \langle m: \text{nat} \rightarrow \text{\_ shift n right m places; floor} (n \times 2^m) \text{ or div} n (2^m) \rangle \text{result} \text{r: \text{nat}:=} n [\text{for i:= 0;} \text{..m [r:= div r} \times 2]]\rangle\rangle.

\text{new} \text{gcd} = \langle a: \text{(nat+1)} \rightarrow \langle b: \text{(nat+1)} \rightarrow \text{\_ greatest common divisor of a and b} \\
a=b \vdash a = a \times b \vdash \text{gcd} a (b-a) \vdash \text{gcd} (a-b) b)\rangle.

\text{new} \text{norm} [\text{plan} \text{num:: nat+1} [\text{plan} \text{denom:: nat+1} ` normalize \text{num/denom} \\
[\text{new} \text{g:= gcd num denom. num:= num/g. denom:= denom/g}]\rangle\rangle.

\text{new} \text{count: nat:= 0. \_ number of examples}
\text{new} \text{glen: nat:= 0. \_ total length of quote representations}
\text{new} \text{rlen: nat:= 0. \_ total length of numerator/denominator representations}

\text{for} \text{length:= 1;} \text{..15}
[\text{for} \text{string:= 0;} \text{..(shl 1 length) \_ each string of that length}
[\text{for} \text{quote:= 0;} \text{..length \_ each quote position (at least one bit to left of quote) \_ if even (shr string (length–1)) \_ even (shr string (quote–1)) \_ roll-normalized}
⟦ if ` repeat-normalized
result repeatnorm: bin:= ⊤
[ new len: nat:= div (length–quote) 2. ` the length of the possibly repeating part
trythislen [ if len>0 ` 1 ≤ len ≤ (length–quote)/2
[ new extract = \{i: nat → \langle l: nat → ` index i length l
shr string i – shl (shr string (i+l)) l \}\}.
new ex:= extract quote len.
if ` the negative part is a repetition (twice or more) of ex
result r: bin:= ⊤
[ new i: nat:= quote+len. ` i+len ≤ length
iloop [ new ey:= extract i len.
if ex=ey [ i:= i+len. ` i≤len
[ if i+len ≤ length [ iloop ]
else [ r:= ⊥ ]
[ for point:= 0;..length+1 ` each point position (right end, interior, left end)
[ if ` the rightmost bit is 1 or it's to the left of quote or point
odd string v (quote=0) v (point=0)
[ ` convert to numerator/denominator
if num<0 [ num:= –num ]
new denom: nat:= shl (shr 1 (length–quote) – 1) point.
norm num denom.
` update statistics
count:= count+1. qlen:= qlen+length.
rlen:= rlen+1. ` for the sign
loop [ num:= div num 2. rlen:= rlen+1.
if num>0 [ loop ]]
loop [ denom:= div denom 2. rlen:= rlen+1.
! “In “; count; “ examples, quote average length = “;
qlen/count; “, num/denom average length = “; rlen/count.

old shl. old shr. old gcd. old norm. old count. old qlen. old rlen

Huffman Codes

new Huffman ` a program to compute Huffman minimum redundancy prefix codes
[ ` input: keys
` output: screen
` use: back find nat nil nl point text unquote

new tree = [text], [tree; tree]. ` a binary tree with texts at the leaves
new forest: *[nat; tree]:= nil. ` the data structure. A string of trees, with a frequency for each tree

inputstart
[ ! “Enter a frequency, then a colon, then a message, then a new line, and repeat. ”;
“ To end, just enter a new line.”; nl.
readloop
if ↔?? = 0 ` Just new line was pressed.
[if forest = 0 `We haven't had any input yet. We need at least one
[! “Insufficient input. Try again.”. inputstart].
new c:= find “:” ??.
if c = ↔?? ![“Bad format: no colon. Try again.”. readloop].
new freq:= unquote (?!(0;..c)).
if freq=“error” ![“Bad frequency format. Try again.”. readloop].
new message:= ??(c;..↔??).
`
find where the new data goes in forest and put it there.
new i:= 0.
fi findloop ![if i = ↔forest v (freq ≤ (forest\0)0) ` found where it goes
[forest:= forest\0;..i]; [freq; [message]]; forest\(i;..↔forest)
else ![i:= i+1. findloop]].
`
` forest is now a nonempty string of pairs, each pair consisting of a frequency and a tree, each
` tree is a single leaf, each leaf is a list-text. They are in non-decreasing frequency order.
` For example: [3; [“a”]]; [4; [“b”]]; [9; [“c”]]; [12; [“d”]]; [15; [“e”]]; [20; [“f”]]
new here: nat:= 0. ` A new tree must be moved to position here or later.
loop ![if ↔forest ≥ 2
[` combine the first two trees into a new tree t
new t:= [(forest\0)0 + (forest\1)0; [(forest\0)1 ; (forest\1)1]].
` remove those first two trees from the forest
forest:= forest\(2;..↔forest).
` put tree t into its place in the forest
innerloop ![if here = ↔forest v (t 0 < (forest\here)0) ` we've found where it goes
[forest:= forest\0;..here]; t; forest\(here;..↔forest). loop]
else ![here:= here+1. innerloop]].
`
` forest is now a single pair consisting of the total of all frequencies and a code tree.
new t:= forest\1. ` the code tree
` Walk the tree, depth-first, printing leaves and their codes
new p: point t:= nil. ` a path within t starting at the root
new pt: text:= “.”. ` same path as p but as a text for printing
loop ![if ~(t p): text ` we are at a leaf
[! “code:”; pt: “; message:”; ~(t p); nl]

Read Password

` program to read a password, allowing corrections, displaying blobs
` input: keys
` output: screen
` use: char delete nl text textid
new password: text: “”.
pswd ![“Please enter password followed by return: ”.
read ![? char.
if ??=nl [if password=“” [!“Empty password. Try again.”; nl. pswd]
else ["nl"]]
else [if ??=delete [if password=“” [password:= password\(0;..\leftrightarrow password–1\). ‘delete’]]
else [password:= password; ??. !“•”].
read]]].
new identity:= textid password. old password

Grammars

LL(1) Grammar

In this grammar, for each nonterminal, every production except possibly the last begins with a different terminal. So director sets are not needed, and that's a special case of LL(1) that deserves its own name; perhaps LL(1/2). To parse a program, the parse stack begins with only the program nonterminal on it, and ends empty with no more input. However, ProTem functions as an operating system, parsing and executing each sequent in turn. So the parse stack begins with sequent on top, and . below it. When the stack is empty, the sequent is executed, the parse stack is reinitialized, and parsing resumes. A name control program is responsible for classifying names. For efficiency, the productions (except possibly the last) for each nonterminal should be placed in order of frequency. The following nonterminals can be eliminated by replacing them with their one production: program sequent data data6 data5 data4 data3 data1. This leaves the grammar with \(31–8 = 23\) nonterminals.

program sequent moreprogram

moreprogram . program

empty

sequent phrase moresequent

moresequent || sequent

empty

phrase new simplename afternewname

old simplename compounder

[ program ]

if data [ program ] elsepart

case data [ program ] elsepart

for simplename := data [ program ]

plan simplename parameterkind data [ program ] arguments

! data

? inputafterq

simplename aftersimplename

afternewname : data := data

= data

:= data

? data ! data

[ program ]

_ afterunderscore

#1
simplename compounder
empty

afterunderscore simplename afternewname
empty

compounder _ simplename compounder
empty

elsepart else [ program ]
empty

parameterkind :
::
!
?

 aftersimplename [ program ]
compounder aftername

aftername := data
! data
? inputafterq
arguments

inputafterq ! echo
data afterpattern

afterpattern ! echo
empty

echo simplename compounder
empty

arguments number arguments
∞ arguments
text arguments
⊤ arguments
⊥ arguments
result simplename : data := data [ program ] arguments
{ data } arguments
[ data ] arguments
( data ) arguments
( simpname : data0 → data ) arguments
simpname compounder arguments
?? arguments
!! arguments
empty

data data6 moredata
moredata ⊨ data = data6 moredata
empty

data6 data5 moredata6

moredata6 = data5 moredata6
≠ data5 moredata6
< data5 moredata6
> data5 moredata6
≤ data5 moredata6
≥ data5 moredata6
: data5 moredata6
empty

data5 data4 moredata5

moredata5 , data4 moredata5
... data4 moredata5
l data4 moredata5
<= data = data4 moredata5
empty

data4 data3 moredata4

moredata4 + data3 moredata4
− data3 moredata4
;; data3 moredata4
; data3 moredata4
;.. data3 moredata4
' data3 moredata4
empty

data3 data2 moredata3

moredata3 × data2 moredata3
/ data2 moredata3
∧ data2 moredata3
v data2 moredata3
empty

data2 # data2
− data2
~ data2
+ data2
□ data2
◊ data2
\ data2
* data2
¢ data3
c data3
↔ data2
data1 moredata2

moredata2
  * data2 moredata2
  \ data2 moredata2
  → data2 moredata2
  ^ data2 moredata2
  ^^ data2 moredata2
  empty

data1 data0 moredata1

moredata1
  % moredata1
  @ data0 moredata1
  & data0 moredata1
  arguments

data0
  number
  ∞
  text
  ⊤
  ⊥
  result simplename : data := data [ program ]
  { data }
  [ data ]
  ( data )
  ⟨ simplename : data0 → data ⟩
  simplename compounder
  ??
  !!

LR(0) Grammar

The following grammar has no reduce-reduce choices and no shift-reduce choices. It has shift-shift choices. Such a grammar is commonly called LR(0), but it shouldn't be, because a shift action pushes an input symbol onto the parse stack, and therefore a shift action depends on the input symbol. It is a special case of LR(1) that deserves its own name, but not LR(0); perhaps LR(1/2). To parse a program, the parse stack begins empty, and ends with only the program nonterminal on it and no more input. However, ProTem functions as an operating system, parsing and executing each sequent in turn. So the parse stack begins empty, and ends with . on top and sequent below it. The sequent is executed, the parse stack is reinitialized, and parsing resumes. A name control program is responsible for classifying names.

program sequent
  program . sequent

sequent phrase
  sequent || phrase

phrase
  new name : data := data
  new name := data
new name = data
new name [ program ]
new name ? data ! data
new name #1
new name _
new name name
old name
name := data
name ! data
! data
name ? data
? data
name ? data ! name
? data ! name
name ? data !
? data!
name ? ! name
? ! name
name ? !
? !
simplename [ program ]
if data [ program ]
if data [ program ] else [ program ]
case data [ program ]
case data [ program ] else [ program ]
for simplename := data [ program ]
[ program ]
plan

plan simplename : data [ program ]
plan simplename :: data [ program ]
plan simplename ? data [ program ]
plan simplename ! data [ program ]
plan data0
name

data
data6 ⊨ data = data

data6
data6 = data5
data6 ≠ data5
data6 < data5
data6 > data5
data6 ≤ data5
data6 ≥ data5
data6 : data5
data5

data5
data5 , data4
data5 .. data4
data5 | data4
data5 ↔ data4

data4
data4 ; data3
data4 ;.. data3
data4 ;; data3
data4 * data3
data4 + data3
data4 – data3
data3

data3
data3 × data2
data3 / data2
data3 ∧ data2
data3 ∨ data2
data2

data2
+ data2
− data2
e data2
↔ data2
# data2
~ data2
□ data2
$ data2
* data2
data1 * data2
data1 → data2
data1 ^ data2
data1 \ data2
data1 ^^ data2
data1

data1
data1 data0
data1 @ data0
data1 %
data1 & data0
name ??
name !
data0

data0
number
∞
text
⊤
⊥
[ data ]
{ data }
\begin{verbatim}
( data )
( simplename : data0 \mapsto data )
result simplename : data := data [ program ]
??
!!
name

name simplename
name _ simplename
\end{verbatim}

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The first public mention of ProTem was


Theo Norvell wrote an MSc thesis in 1988 titled “Expressions, Types, and Data Structures in ProTem”. Hugh Redelmeier acted as design consultant and critic in 1990. Brian Parkinson found a bug in the implementation in 1990. The design of ProTem has been improved since then, and the old implementation is now out-of-date. A new implementation is partly written.