History of my Problems with the Halting Problem
2013 August 14, 2014 July 6, 2022 April 15

In 1986, during a lecture at the Marktoberdorf Summer School, I dared to suggest that there are serious flaws in Turing's proof of the incomputability of the halting function. I have never been invited back to the Marktoberdorf Summer School. I made the same comments at a seminar at my own Department of Computer Science at the University of Toronto the next year. The reaction from two of my department's best theoreticians was immediate and angry. Wanting to continue my career there, I decided I should drop the subject. But in 1989 I wrote a paper detailing the problems with Turing's proof, and submitted it to the journal *Formal Aspects of Computing*. The referee's report began as follows:

> I cannot recommend the present article for publication in any professional journal. It falls far below minimal professional standards in logic or philosophy and does not contain anything of interest concerning computing. My general impression is that this is the kind of paper a bright high school student might write after reading Hofstadter's book. If so, he should be encouraged to get some real knowledge of the subject, but not to try to get papers of this sort published. Some comments on the contents are given below, but I believe that a detailed commentary can only be profitably made face-to-face by a competent teacher.

The funniest part of this review is that I was an editor of the journal, with my name and address printed on the back of every issue. The handling editor, a friend of mine, was apologetic for the tone of the review, but he did reject the paper. I did not wish to gain a reputation as a crank and hurt the rest of my career, so this time I did drop the subject.

I picked up the subject again in 2008, just 4 years before retirement, when I cared more about the validity of the foundations of computer science than I did about hurting my reputation or career. I rewrote the paper, and submitted it to the *Journal of the Association for Computing Machinery*. Here is the only review, in its entirety.

> This paper argues against well-established results of Turing. The arguments are philosophical and not particularly good ones at that, ignoring the mathematical formulations that make the theorems true. The author might not like the fact that the halting function is incomputable, but it is and the author can't wish away that fact, or Turing's incomplete sets anymore than he can wish away gravity.

The arguments in my paper are entirely technical and mathematical. And I believe Einstein did "wish away gravity", turning it into geometry, in general relativity. The editor Lance Fortnow then banned me from further submission to *JACM*. I have published in *JACM* before, and I don't need to publish there again.

What strikes me most about these reviews is that they do not point out any error in my arguments and proofs. They point out, with accompanying insults, that I am making a claim that is contrary to the current orthodoxy. I know that. They <u>know</u> that Turing proved that the Halting Problem is incomputable; it's in all the textbooks. So they <u>know</u> from my paper's abstract that the paper is wrong. So they feel no need to read my arguments carefully.

Another tactic was taken by *Theoretical Computer Science*, who said "This paper is totally out of the scope of TCS.". There couldn't be a paper more in the scope of this journal. Similarly, on the same day the paper was submitted to the *International Journal of Computer Science*, its editor E.Judas said "Your paper submission is not an interest of IJCS!" (his exclamation mark). The journal says "IJCS covers all areas of computer science.", and lists theories first.

I sent the "Reconstructing the Halting Problem" version of the paper to CiE2014, and received the most negative score –3 (reject (even for presentation)) with the highest confidence 5 (expert). One review, in its entirety, said

> As best as I can make out, the author's claim boils down to quibbling that the diagonal argument still goes through when the word "terminate" is replaced with an undefined term. As his own allusion to Rice's Theorem spells out, this is in no way damaging to the argument, and almost any predicate will work just as well. This paper hinges on basic misunderstandings and should not be considered for publication.

Somehow, the referee missed the main point: that the specification of halting is inconsistent. The other, in its entirety, said

> I think the source of the problem is that the author definitely has not thoroughly understood the halting problem, the definition and the incomputability proof. The proof starts with the assumption that if halting is computable, then we can construct another computable function that does not exist at all! (please see a poetic undecidability proof for halting problem http://www.cs.rice.edu/~vardi/comp409/scooping.pdf). And the essence of the proof is self-reference. However, the author may misunderstand the self-reference, thinking that it is inherently inconsistent and thus claiming that the famous proof is wrong.

I commend the referees for staying technical, but their reading was cursory and their reviews shallow. The second referee informs me that the essence of the proof is self-reference (as if I had missed that point), and wonders if I think self-reference is inherently inconsistent. As should be obvious from the paper, self-reference is not inherently inconsistent, but the one in the proof is inconsistent; indeed, it is constructed for the purpose of being inconsistent.

As I approached retirement, I spoke about my ideas on the incomputability of the Halting Problem as an invited speaker at a retrospective conference. Either in my audience, or reading the proceedings, were three Dutch computer scientists: Ruurd Kuiper, Tom Verhoeff, and Kees Huizing. They told me that they had been in my audience at the Marktoberdorf Summer School, as students, twenty years earlier. They then wrote a paper that they thought remedied all deficiencies. I was sent their paper for refereeing. I was thrilled that someone was replying technically to my technical arguments. Although they did not remedy all deficiencies, it seemed to me that they made some interesting and valid arguments, so I recommended acceptance. I saw them again at the Turing100 conference in Manchester in 2012 and we discussed our differences. They then made a suggestion for improving my presentation. This is how science should be conducted.

I was invited by Olga Tveretina of the Karlsruhe Institute of Technology to present my paper at COMPUTING 2011 (Symposium on 75 years of Turing Machine and Lambda-

Calculus), in Karlsruhe on 2011 October 21. Some eminent computer scientists were in attendance, and it seemed to me from comments I received that the presentation went well.

I gave a talk on the Halting Problem on my final day of work, 2012 June 12, as part of my retirement celebration. The talk is recorded and on my website. I begin that talk by singing (in 2012) the poem that the CiE2014 referee tried to tell me (in 2014) is a reasonable proof of incomputability.

I gave more-or-less the same talk at the University of Waterloo on 2013 November 28. To everyone's amazement, the talk was interrupted by an attempted filibuster! I devote a separate piece to the description of that and its surrounding events.

I now have three publication on the subject.

> E.C.R.Hehner: "Problems with the Halting Problem", *Advances in Computer Science and Engineering*, v.10 n.1 p.31-60, 2013,
> http://www.cs.utoronto.ca/~hehner/PHP.pdf

> E.C.R.Hehner: "Objective and Subjective Specifications", WST Workshop on Termination, Oxford, 2018 July 18, www.cs.utoronto.ca/~hehner/OSS.pdf

> E.C.R.Hehner: "Epimenides, Gödel, Turing: an Eternal Gölden Twist", SN Computer Science v.1 p.308, 2020 September
> www.cs.utoronto.ca/~hehner/EGT+.pdf

There is an archive, named arXiv, hosted by Cornell University, for mathematical and technical papers. It is not refereed, and not considered as publication for academic purposes, but it is moderated to ensure that the submissions are serious and in its range of topics. Its purpose is to timestamp research so that a researcher can prove priority. I am not interested in priority, but I am interested in having the ideas permanently recorded. So I sent the paper

> E.C.R.Hehner: "Halting, the Power of Mathematics, and Religion",
> 2020 September 20, www.cs.utoronto.ca/~hehner/HPMR.pdf

to arXiv. It was instantly rejected. The paper makes an analogy between the halting problem and religion, and I speculate that the rejection is due to fear that my paper may offend religious people, even though there is nothing offensive in the paper.

There really is a serious flaw in the proofs of the incomputability of the Halting Problem. Since people seem to prefer the argument from authority over the reasoned arguments in my paper, my hope is that sometime, maybe after my death, someone with authority will read my reasoned arguments, and give them authority. Or maybe someone with authority will discover the same thing I have discovered. Either way, I would like the theoretical foundation of computing to be corrected.

my papers on halting