

# the Gambler and the Mathematician

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A gambler walked up to a mathematician and said “Let's make a bet. You give me \$100 now, and if a certain event happens within a year I'll give you \$500,000. I'm betting that it won't happen. Are you willing to bet that it will?”. The mathematician was intrigued; losing \$100 wouldn't hurt much, and gaining \$500,000 would be great. “What event?”, she asked. She had three purposes in asking this question. First, she needed to be sure that it's an event the gambler cannot prevent. Second, its occurrence has to be clear and knowable to both of them, so they will agree on whether it has occurred. And finally, she needed to know its probability. If its probability is better than 1 in 5,000 then it's a good bet for her, and she should agree to it. If its probability is worse than 1 in 5,000 then it's a bad bet for her, and she should not agree to it.

A standard choice of event used by lotteries is that a particular sequence of decimal digits, chosen in advance by the player, is displayed when a sequence of balls drops, each one displaying one digit. (There may be a variety of winning sequences or partial sequences, but let's keep it simple.) This event cannot be influenced by anyone, everyone can see what the winning sequence is, and it's easy to calculate the probability. If there are 3 or fewer balls in the sequence, the probability is 1 in 1,000 or better, and the mathematician should play. If there are 4 or more balls in the sequence, the probability is 1 in 10,000 or worse, and the mathematician should not play. But this is not the event that the gambler had in mind.

The gambler said “I'll give you a choice of events. If you like, the event can be that your house burns down sometime in the coming year.”. The mathematician thought “That's a gruesome, unpleasant choice of event, but that's not really relevant to whether I should bet. The gambler cannot prevent it, and it's clear whether it has happened. I just need to know its probability.”. The gambler continued: “I have collected all the fire statistics for your neighborhood, and the probability that your house will burn down next year is 1 in 10,000.”. The mathematician immediately saw that this probability does not give her a fair chance of winning, so she declined to bet. “What's my other choice?”, she asked. “The other choice is that you win the Field's medal.” That's one of the highest honors for a mathematician, a delightfully pleasant choice of event, but again, the pleasantness of the event is irrelevant. She knew, and apparently the gambler did not know, that she was among the best thousand mathematicians in the world. Not knowing where among the best thousand mathematicians she stood, she estimated her chance at about 1 in 1,000, and agreed to the bet.

A mathematician is different from ordinary people; ordinary people do not understand mathematics. An ordinary person usually agrees to bet that their house will burn down, even when it's a bad bet for them. The \$500,000 is called insurance, the gambler is called an insurance agent, and the \$100 is called the premium. A typical insurance agent has no idea what the probability is for the event that is being insured against, even though the customer needs that piece of information in order to decide whether to buy the insurance. And even if the agent knew that information, a typical customer wouldn't know what to do with it. But the insurance company does know the probability, and the fact that insurance companies make a profit is the evidence that the probabilities favor them, not their customers.

The agent tells the customer “We will protect your house against fire.”; that's a lie; the insurance company cannot prevent fire. Or maybe the agent says “We will protect you against loss due to fire.”; that's a bit better, but still false; the insurance company cannot prevent the loss of your house or belongings. All they can do is pay off their bet.

With this story, I have tried to show that fire insurance is a bet you make with an insurance company. Similarly all other insurance is a bet. To decide whether to bet (buy

insurance), you need to know the buy-in (premium), payoff (amount of insurance), and the probability. That's all that's relevant. Whether the event is unpleasant (house burns down), pleasant (win Field's medal), or neutral (the falling balls showed my sequence of digits) is irrelevant for making a rational, mathematically sound decision. But there is a psychological reason why unpleasant events are offered as payoff triggers: it's nice to balance a bad day (house burns down) with a good day (winning a large bet). Apparently this psychological fact outweighs mathematics, especially for that large part of the population that can't do math.

Some will rebut: It's not just "psychological", and it's not just "nice"; I can't afford to lose my house without also winning a compensating bet. That's a good point, but it doesn't justify making a bad bet. Not all insurance is a bad deal. Not-for-profit insurance, sometimes run by government, sometimes run as a co-operative, can be a good deal. Self-insurance, otherwise known as saving for a rainy day, is a good deal. But buying insurance from a profit-making insurance company is a bad deal, and the more profitable the company, the worse the deal.

There is a portion of the population that is mostly rational, maybe even knows enough math, but cannot break free from the constant psychological bombardment from insurance companies. The companies say "Don't take a chance; buy insurance." when actually buying insurance is gambling. They invoke the fear and sadness of the unpleasant event to keep you from making a rational decision. And they invoke some questionable mathematics. An example is "nonlinear utility": your first dollar is worth more to you than your 500,000th dollar. Your first dollars keep you alive, and your last dollars just add marginally to your comfort. You need to protect your ability to live (by keeping a roof over your head), and therefore, they conclude, it is worth buying insurance even if the probability of "winning" the payoff doesn't justify paying the premium from a purely gambling point of view. But this argument makes no sense. If you have very little money, then you can't afford to be giving it to an insurance company. If you have lots of money, you don't need the insurance. Nothing justifies making a bad bet.