Quantifiers

A quantifier is an operator that applies to a function.

It is defined from a two-operand symmetric associative operator.
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\( \forall p \) is defined from \( \land \) “for all”
A quantifier is an operator that applies to a function.

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\[ \forall p \text{ is defined from } \wedge \text{ “for all” } \forall \langle r: \text{rat} \cdot r \geq 0 \rangle = \bot \]
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Σf is defined from + “sum of”
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\[ \exists p \text{ is defined from } \lor \quad \text{“there exists”} \quad \exists \langle n: \text{nat} \cdot n = 0 \rangle = \top \]

\[ \Sigma f \text{ is defined from } + \quad \text{“sum of”} \quad \Sigma \langle n: \text{nat}+1 \cdot 1/2^n \rangle = 1 \]
Quantifiers

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\[ \forall p \text{ is defined from } \land \quad "\text{for all}" \quad \forall \langle r : \text{rat} \cdot r \geq 0 \rangle = \bot \]

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A quantifier is an operator that applies to a function.

It is defined from a two-operand symmetric associative operator.

\( \forall p \) is defined from \( \land \) “for all” \( \forall \langle r: \text{rat} \cdot r \geq 0 \rangle = \bot \)

\( \exists p \) is defined from \( \lor \) “there exists” \( \exists \langle n: \text{nat} \cdot n = 0 \rangle = \top \)

\( \Sigma f \) is defined from \( + \) “sum of” \( \Sigma \langle n: \text{nat}+1 \cdot \frac{1}{2^n} \rangle = 1 \)

\( \Pi f \) is defined from \( \times \) “product of” \( \Pi \langle n: \text{nat}+1 \cdot \frac{4 \times n^2}{4 \times n^2 - 1} \rangle = \frac{\pi}{2} \)
A quantifier is an operator that applies to a function.

It is defined from a two-operand symmetric associative operator.

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It is defined from a two-operand symmetric associative operator.

\[ \forall p \text{ is defined from } \land \quad \forall \langle r: \text{rat} \cdot r \geq 0 \rangle = \bot \]

\[ \exists p \text{ is defined from } \lor \quad \exists \langle n: \text{nat} \cdot n = 0 \rangle = \top \]

\[ \Sigma f \text{ is defined from } + \quad \Sigma \langle n: \text{nat} + 1 \cdot 1/2^n \rangle = 1 \]

\[ \Pi f \text{ is defined from } \times \quad \Pi \langle n: \text{nat} + 1 \cdot (4 \times n^2)/(4 \times n^2 - 1) \rangle = \pi/2 \]

\[ \uparrow f \text{ is defined from } \uparrow \quad \uparrow \langle x: \text{rat} \cdot x \times (4 - x) \rangle = 4 \]
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\( \Sigma f \) is defined from + “sum of” \( \Sigma \langle n: \text{nat}+1 \cdot 1/2^n \rangle = 1 \)

\( \Pi f \) is defined from \( \times \) “product of” \( \Pi \langle n: \text{nat}+1 \cdot (4 \times n^2)/(4 \times n^2 - 1) \rangle = \pi/2 \)

⇑f is defined from ↑ “maximum of” \( \uparrow \langle x: \text{rat} \cdot x \times (4-x) \rangle = 4 \)

⇓f is defined from ↓ “minimum of”
**Quantifiers**

A quantifier is an operator that applies to a function.

It is defined from a two-operand symmetric associative operator.

\[ \forall p \text{ is defined from } \wedge \text{ “for all” } \]  \[ \forall \langle r: \text{rat} \cdot r \geq 0 \rangle = \bot \]

\[ \exists p \text{ is defined from } \lor \text{ “there exists” } \]  \[ \exists \langle n: \text{nat} \cdot n = 0 \rangle = \top \]

\[ \Sigma f \text{ is defined from } + \text{ “sum of” } \]  \[ \Sigma \langle n: \text{nat}+1 \cdot 1/2^n \rangle = 1 \]

\[ \Pi f \text{ is defined from } \times \text{ “product of” } \]  \[ \Pi \langle n: \text{nat}+1 \cdot (4n^2)/(4n^2 - 1) \rangle = \pi/2 \]

\[ \uparrow f \text{ is defined from } \uparrow \text{ “maximum of” } \]  \[ \uparrow \langle x: \text{rat} \cdot x \times (4-x) \rangle = 4 \]

\[ \downarrow f \text{ is defined from } \downarrow \text{ “minimum of” } \]  \[ \downarrow \langle n: \text{nat}+1 \cdot 1/n \rangle = 0 \]
Quantifiers

abbreviations

\[ \forall r: \text{rat} \cdot r \geq 0 \] abbreviates \[ \forall \langle r: \text{rat} \cdot r \geq 0 \rangle \]

\[ \Sigma n: \text{nat} + 1 \cdot \frac{1}{2^n} \] abbreviates \[ \Sigma \langle n: \text{nat} + 1 \cdot \frac{1}{2^n} \rangle \]
Quantifiers

abbreviations

∀\: \text{rat} \cdot r \geq 0 \quad \text{abbreviates} \quad \forall \langle r: \text{rat} \cdot r \geq 0 \rangle

Σn: nat+1 \cdot \frac{1}{2^n} \quad \text{abbreviates} \quad Σ\langle n: nat+1 \cdot \frac{1}{2^n} \rangle

∀x, y: \text{rat} \cdot x = y+1 \implies x > y \quad \text{abbreviates} \quad ∀x: \text{rat} \cdot ∀y: \text{rat} \cdot x = y+1 \implies x > y

Σn, m: 0..10 \cdot n \times m \quad \text{abbreviates} \quad Σn: 0..10 \cdot Σm: 0..10 \cdot n \times m
∀v: null· b = ⊤
∃v: null· b = ⊥

∀v: x· b = ⟨v: x· b⟩ x
∃v: x· b = ⟨v: x· b⟩ x

∀v: A,B· b = (∀v: A· b) ∧ (∀v: B· b)
∃v: A,B· b = (∃v: A· b) ∨ (∃v: B· b)

Σv: null· n = 0
Σv: x· n = ⟨v: x· n⟩ x
(Σv: A,B· n) + (Σv: A· B· n) = (Σv: A· n) + (Σv: B· n)

Πv: null· n = 1
Πv: x· n = ⟨v: x· n⟩ x
(Πv: A,B· n) × (Πv: A· B· n) = (Πv: A· n) × (Πv: B· n)

⇑v: null· b = –∞
⇓v: null· b = ∞

⇑v: x· b = ⟨v: x· b⟩ x
⇓v: x· b = ⟨v: x· b⟩ x

⇑v: A,B· b = (⇑v: A· b) ↑ (⇑v: B· b)
⇓v: A,B· b = (⇓v: A· b) ↓ (⇓v: B· b)
∀v: null·b = ⊤ \leftarrow

∀v: x·b = ⟨v: x·b⟩x

∀v: A,B·b = (∀v: A·b) ∧ (∀v: B·b)

∃v: null·b = ⊥ \leftarrow

∃v: x·b = ⟨v: x·b⟩x

∃v: A,B·b = (∃v: A·b) ∨ (∃v: B·b)

Σv: null·n = 0 \leftarrow

Σv: x·n = ⟨v: x·n⟩x

(Σv: A,B·n) + (Σv: A·B·n) = (Σv: A·n) + (Σv: B·n)

Πv: null·n = 1 \leftarrow

Πv: x·n = ⟨v: x·n⟩x

(Πv: A,B·n) × (Πv: A·B·n) = (Πv: A·n) × (Πv: B·n)

⇑v: null·b = –∞ \leftarrow

⇑v: x·b = ⟨v: x·b⟩x

⇑v: A,B·b = (⇑v: A·b) ↑ (⇑v: B·b)

⇑v: A,B·b = (⇑v: A·b) ↑ (⇑v: B·b)

⇓v: null·b = ∞ \leftarrow

⇓v: x·b = ⟨v: x·b⟩x

⇓v: A,B·b = (⇓v: A·b) ↓ (⇓v: B·b)

⇓v: A,B·b = (⇓v: A·b) ↓ (⇓v: B·b)
\[ \forall v: \text{null} \cdot b = \top \]
\[ \exists v: \text{null} \cdot b = \bot \]
\[ \forall v: x \cdot b = \langle v: x \cdot b \rangle x \]
\[ \exists v: x \cdot b = \langle v: x \cdot b \rangle x \]
\[ \forall v: A,B \cdot b = (\forall v: A \cdot b) \land (\forall v: B \cdot b) \]
\[ \exists v: A,B \cdot b = (\exists v: A \cdot b) \lor (\exists v: B \cdot b) \]

\[ \Sigma v: \text{null} \cdot n = 0 \quad \text{because } x + 0 = x \]
\[ \Sigma v: x \cdot n = \langle v: x \cdot n \rangle x \]
\[ (\Sigma v: A,B \cdot n) + (\Sigma v: A' B \cdot n) = (\Sigma v: A \cdot n) + (\Sigma v: B \cdot n) \]

\[ \Pi v: \text{null} \cdot n = 1 \]
\[ \Pi v: x \cdot n = \langle v: x \cdot n \rangle x \]
\[ (\Pi v: A,B \cdot n) \times (\Pi v: A' B \cdot n) = (\Pi v: A \cdot n) \times (\Pi v: B \cdot n) \]

\[ \uparrow v: \text{null} \cdot b = -\infty \]
\[ \downarrow v: \text{null} \cdot b = \infty \]
\[ \uparrow v: x \cdot b = \langle v: x \cdot b \rangle x \]
\[ \downarrow v: x \cdot b = \langle v: x \cdot b \rangle x \]
\[ \uparrow v: A,B \cdot b = (\uparrow v: A \cdot b) \uparrow (\uparrow v: B \cdot b) \]
\[ \downarrow v: A,B \cdot b = (\downarrow v: A \cdot b) \downarrow (\downarrow v: B \cdot b) \]
∀v: null· b = ⊤
∀v: x· b = ⟨v: x· b⟩ x
∀v: A·B· b = (∀v: A· b) ∧ (∀v: B· b)
∃v: null· b = ⊥
∃v: x· b = ⟨v: x· b⟩ x
∃v: A·B· b = (∃v: A· b) ∨ (∃v: B· b)

Σv: null· n = 0
Σv: x· n = ⟨v: x· n⟩ x
(Σv: A,B· n) + (Σv: A·B· n) = (Σv: A· n) + (Σv: B· n)

Πv: null· n = 1 ← because x×1 = x
Πv: x· n = ⟨v: x· n⟩ x
(Πv: A,B· n) × (Πv: A·B· n) = (Πv: A· n) × (Πv: B· n)

⇑v: null· b = –∞
⇑v: x· b = ⟨v: x· b⟩ x
⇑v: A,B· b = (⇑v: A· b) ↑ (⇑v: B· b)
⇓v: null· b = ∞
⇓v: x· b = ⟨v: x· b⟩ x
⇓v: A,B· b = (⇓v: A· b) ↓ (⇓v: B· b)
∀ \(v: \text{null} \cdot b = \top\)

∀ \(v: x \cdot b = \langle v: x \cdot b \rangle x\)

∀ \(v: A, B \cdot b = (\forall v: A \cdot b) \land (\forall v: B \cdot b)\)

∃ \(v: \text{null} \cdot b = \bot \) because \(x \lor \bot = x\)

∃ \(v: x \cdot b = \langle v: x \cdot b \rangle x\)

∃ \(v: A, B \cdot b = (\exists v: A \cdot b) \lor (\exists v: B \cdot b)\)

\(\Sigma v: \text{null} \cdot n = 0\)

\(\Sigma v: x \cdot n = \langle v: x \cdot n \rangle x\)

\((\Sigma v: A, B \cdot n) + (\Sigma v: A' B \cdot n) = (\Sigma v: A \cdot n) + (\Sigma v: B \cdot n)\)

\(\Pi v: \text{null} \cdot n = 1\)

\(\Pi v: x \cdot n = \langle v: x \cdot n \rangle x\)

\((\Pi v: A, B \cdot n) \times (\Pi v: A' B \cdot n) = (\Pi v: A \cdot n) \times (\Pi v: B \cdot n)\)

\(\uparrow v: \text{null} \cdot b = -\infty\)

\(\downarrow v: \text{null} \cdot b = \infty\)

\(\uparrow v: x \cdot b = \langle v: x \cdot b \rangle x\)

\(\downarrow v: x \cdot b = \langle v: x \cdot b \rangle x\)

\(\uparrow v: A, B \cdot b = (\uparrow v: A \cdot b) \uparrow (\uparrow v: B \cdot b)\)

\(\downarrow v: A, B \cdot b = (\downarrow v: A \cdot b) \downarrow (\downarrow v: B \cdot b)\)
\[ \forall v: \text{null} \cdot b = \top \leftarrow \text{because } x \land \top = x \]
\[ \exists v: \text{null} \cdot b = \bot \]
\[ \forall v: x \cdot b = \langle v: x \cdot b \rangle x \]
\[ \exists v: x \cdot b = \langle v: x \cdot b \rangle x \]
\[ \forall v: A,B \cdot b = (\forall v: A \cdot b) \land (\forall v: B \cdot b) \]
\[ \exists v: A,B \cdot b = (\exists v: A \cdot b) \lor (\exists v: B \cdot b) \]
\[ \sum v: \text{null} \cdot n = 0 \]
\[ \sum v: x \cdot n = \langle v: x \cdot n \rangle x \]
\[ (\sum v: A,B \cdot n) + (\sum v: A \cdot B \cdot n) = (\sum v: A \cdot n) + (\sum v: B \cdot n) \]
\[ \prod v: \text{null} \cdot n = 1 \]
\[ \prod v: x \cdot n = \langle v: x \cdot n \rangle x \]
\[ (\prod v: A,B \cdot n) \times (\prod v: A \cdot B \cdot n) = (\prod v: A \cdot n) \times (\prod v: B \cdot n) \]
\[ \uparrow v: \text{null} \cdot b = -\infty \]
\[ \downarrow v: \text{null} \cdot b = \infty \]
\[ \uparrow v: x \cdot b = \langle v: x \cdot b \rangle x \]
\[ \downarrow v: x \cdot b = \langle v: x \cdot b \rangle x \]
\[ \uparrow v: A,B \cdot b = (\uparrow v: A \cdot b) \uparrow (\uparrow v: B \cdot b) \]
\[ \downarrow v: A,B \cdot b = (\downarrow v: A \cdot b) \downarrow (\downarrow v: B \cdot b) \]
\( \forall v: \text{null} \cdot b = \top \)
\( \exists v: \text{null} \cdot b = \bot \)
\( \forall v: x \cdot b = \langle v: x \cdot b \rangle x \)
\( \exists v: x \cdot b = \langle v: x \cdot b \rangle x \)
\( \forall v: A.B \cdot b = (\forall v: A \cdot b) \land (\forall v: B \cdot b) \)
\( \exists v: A.B \cdot b = (\exists v: A \cdot b) \lor (\exists v: B \cdot b) \)

\( \Sigma v: \text{null} \cdot n = 0 \)
\( \Sigma v: x \cdot n = \langle v: x \cdot n \rangle x \)
\( (\Sigma v: A.B \cdot n) + (\Sigma v: A^*B^* \cdot n) = (\Sigma v: A \cdot n) + (\Sigma v: B \cdot n) \)

\( \Pi v: \text{null} \cdot n = 1 \)
\( \Pi v: x \cdot n = \langle v: x \cdot n \rangle x \)
\( (\Pi v: A.B \cdot n) \times (\Pi v: A^*B^* \cdot n) = (\Pi v: A \cdot n) \times (\Pi v: B \cdot n) \)

\( \uparrow v: \text{null} \cdot b = -\infty \quad \text{because} \quad x^{\uparrow -\infty} = x \quad \downarrow v: \text{null} \cdot b = \infty \)
\( \uparrow v: x \cdot b = \langle v: x \cdot b \rangle x \)
\( \downarrow v: x \cdot b = \langle v: x \cdot b \rangle x \)
\( \uparrow v: A.B \cdot b = (\uparrow v: A \cdot b) \uparrow (\uparrow v: B \cdot b) \)
\( \downarrow v: A.B \cdot b = (\downarrow v: A \cdot b) \downarrow (\downarrow v: B \cdot b) \)
∀v: null⋅b = ⊤
∀v: x⋅b = ⟨v:x⋅b⟩x
∀v: A,B⋅b = (∀v:A⋅b)∧(∀v:B⋅b)
∃v: null⋅b = ⊥
∃v: x⋅b = ⟨v:x⋅b⟩x
∃v: A,B⋅b = (∃v:A⋅b)∨(∃v:B⋅b)

Σv: null⋅n = 0
Σv: x⋅n = ⟨v:x⋅n⟩x
(Σv:A,B⋅n) + (Σv:A\dot{B}⋅n) = (Σv:A⋅n) + (Σv:B⋅n)

Πv: null⋅n = 1
Πv: x⋅n = ⟨v:x⋅n⟩x
(Πv:A,B⋅n) × (Πv:A\dot{B}⋅n) = (Πv:A⋅n) × (Πv:B⋅n)

⇑v: null⋅b = −∞
⇑v: x⋅b = ⟨v:x⋅b⟩x
⇑v: A,B⋅b = (⇑v:A⋅b)↑(⇑v:B⋅b)
⇓v: null⋅b = ∞ ← because x⇓∞ = x
⇓v: x⋅b = ⟨v:x⋅b⟩x
⇓v: A,B⋅b = (⇓v:A⋅b)↓(⇓v:B⋅b)
\[ \forall v: \text{null} \cdot b = \top \]
\[ \exists v: \text{null} \cdot b = \bot \]
\[ \forall v: x \cdot b = \langle v: x \cdot b \rangle x \]
\[ \exists v: x \cdot b = \langle v: x \cdot b \rangle x \]
\[ \forall v: A,B \cdot b = (\forall v: A \cdot b) \land (\forall v: B \cdot b) \]
\[ \exists v: A,B \cdot b = (\exists v: A \cdot b) \lor (\exists v: B \cdot b) \]
\[ \Sigma v: \text{null} \cdot n = 0 \]
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\[ (\Sigma v: A,B \cdot n) + (\Sigma v: A'B \cdot n) = (\Sigma v: A \cdot n) + (\Sigma v: B \cdot n) \]
\[ \Pi v: \text{null} \cdot n = 1 \]
\[ \Pi v: x \cdot n = \langle v: x \cdot n \rangle x \]
\[ (\Pi v: A,B \cdot n) \times (\Pi v: A'B \cdot n) = (\Pi v: A \cdot n) \times (\Pi v: B \cdot n) \]
\[ \uparrow v: \text{null} \cdot b = -\infty \]
\[ \downarrow v: \text{null} \cdot b = \infty \]
\[ \uparrow v: x \cdot b = \langle v: x \cdot b \rangle x \]
\[ \downarrow v: x \cdot b = \langle v: x \cdot b \rangle x \]
\[ \uparrow v: A,B \cdot b = (\uparrow v: A \cdot b) \uparrow (\uparrow v: B \cdot b) \]
\[ \downarrow v: A,B \cdot b = (\downarrow v: A \cdot b) \downarrow (\downarrow v: B \cdot b) \]
∀ \text{null} \cdot b = \top

∃ \text{null} \cdot b = \bot

∀ v: \text{x} \cdot b = \langle v: \text{x} \cdot b \rangle x

∃ v: \text{x} \cdot b = \langle v: \text{x} \cdot b \rangle x

∀ v: A,B \cdot b = (\forall v: A \cdot b) \land (\forall v: B \cdot b)

∃ v: A,B \cdot b = (\exists v: A \cdot b) \lor (\exists v: B \cdot b)

Σv: \text{null} \cdot n = 0

Σv: \text{x} \cdot n = \langle v: \text{x} \cdot n \rangle x

Σv: A,B \cdot n = Σv: A \cdot n + Σv: B \cdot n

Πv: \text{null} \cdot n = 1

Πv: \text{x} \cdot n = \langle v: \text{x} \cdot n \rangle x

Πv: A,B \cdot n = Πv: A \cdot n \times Πv: B \cdot n

⇑ v: \text{null} \cdot b = -\infty

⇓ v: \text{null} \cdot b = \infty

⇑ v: \text{x} \cdot b = \langle v: \text{x} \cdot b \rangle x

⇓ v: \text{x} \cdot b = \langle v: \text{x} \cdot b \rangle x

⇑ v: A,B \cdot b = (⇑ v: A \cdot b) \uparrow (⇑ v: B \cdot b)

⇓ v: A,B \cdot b = (⇓ v: A \cdot b) \downarrow (⇓ v: B \cdot b)
∀v: null·b = ⊤
∀v: x·b = ⟨v: x·b⟩x
∀v: A,B·b = (∀v: A·b) ∧ (∀v: B·b)
∃v: null·b = ⊥
∃v: x·b = ⟨v: x·b⟩x
∃v: A,B·b = (∃v: A·b) ∨ (∃v: B·b)

Σv: null·n = 0
Σv: x·n = ⟨v: x·n⟩x
(Σv: A,B·n) + (Σv: A⁺B·n) = (Σv: A·n) + (Σv: B·n)←

Πv: null·n = 1
Πv: x·n = ⟨v: x·n⟩x
(Πv: A,B·n) × (Πv: A⁺B·n) = (Πv: A·n) × (Πv: B·n)←

⇑v: null·b = −∞
⇑v: x·b = ⟨v: x·b⟩x
⇑v: A,B·b = (⇑v: A·b) ↑ (⇑v: B·b)
⇓v: null·b = ∞
⇓v: x·b = ⟨v: x·b⟩x
⇓v: A,B·b = (⇓v: A·b) ↓ (⇓v: B·b)
Solution Quantifier

$p$ is the (bunch of) solutions of predicate $p$

$v: \text{null} \cdot b = \text{}$

$v: x \cdot b = \text{}$

$v: A.B \cdot b = \text{}$
Solution Quantifier

$p$ is the (bunch of) solutions of predicate $p$

$v: null \cdot b = null$

$v: x \cdot b =$

$v: A.B \cdot b =$
Solution Quantifier

$p$ is the (bunch of) solutions of predicate $p$

$v: \textit{null} \cdot b = \textit{null}$

$v: x \cdot b = \textbf{if} \langle v: x \cdot b \rangle \ x \ \textbf{then} \ x \ \textbf{else} \ \textit{null} \ \textbf{fi}$

$v: A.B \cdot b =$
Solution Quantifier

$\$p$ is the (bunch of) solutions of predicate $p$

$\$v: \text{null} \cdot b = \text{null}$

$\$v: x \cdot b = \text{if } \langle v: x \cdot b \rangle \ x \text{ then } x \text{ else null fi}$

$\$v: A.B \cdot b = (\$v: A \cdot b), (\$v: B \cdot b)$
Solution Quantifier

$p$ is the (bunch of) solutions of predicate $p$

$v: \text{null} \cdot b = \text{null}$

$v: x \cdot b = \text{if} \langle v: x \cdot b \rangle x \text{ then } x \text{ else } \text{null} \text{ fi}$

$v: A.B \cdot b = (v: A \cdot b), (v: B \cdot b)$

$i: \text{int} \cdot i^2=4$
Solution Quantifier

§$p$ is the (bunch of) solutions of predicate $p$

§$v$: $null \cdot b = null$

§$v$: $x \cdot b =$ if $\langle v: x \cdot b \rangle x$ then $x$ else $null$ fi

§$v$: $A B \cdot b = (§v: A \cdot b), (§v: B \cdot b)$

§$i$: $int \cdot i^2=4 = -2, 2$
Solution Quantifier

$p$ is the (bunch of) solutions of predicate $p$

$v: \text{null} \cdot b = \text{null}$

$v: x \cdot b = \text{if } \langle v: x \cdot b \rangle x \text{ then } x \text{ else null fi}$

$v: A.\!\!\!\!\!\!B \cdot b = (v: A \cdot b), (v: B \cdot b)$

$i: \text{int} \cdot i^2 = 4 = -2, 2$

$n: \text{nat} \cdot n < 3$
Solution Quantifier

\[ \$p \] is the (bunch of) solutions of predicate \( p \)

\[ \$v: \text{null} \cdot b = \text{null} \]

\[ \$v: x \cdot b = \text{if} \langle v: x \cdot b \rangle x \text{ then } x \text{ else } \text{null} \text{ fi} \]

\[ \$v: A,B \cdot b = (\$v: A \cdot b), (\$v: B \cdot b) \]

\[ \$i: \text{int} \cdot i^2=4 = -2, 2 \]

\[ \$n: \text{nat} \cdot n<3 = 0,..3 \]
Solution Quantifier

\( \$p \) is the (bunch of) solutions of predicate \( p \)

\( \$v: \text{null} \cdot b = \text{null} \)

\( \$v: x \cdot b = \text{if} \langle v: x \cdot b \rangle x \text{ then } x \text{ else } \text{null} \text{ fi} \)

\( \$v: A,B \cdot b = (\$v: A \cdot b), (\$v: B \cdot b) \)

\( \{ \$i: \text{int} \cdot i^2=4 \} = \{-2, 2\} \)

\( \{ \$n: \text{nat} \cdot n<3 \} = \{0,..3\} \)
Solution Quantifier

$p$ is the (bunch of) solutions of predicate $p$

$v: \text{null} \cdot b = \text{null}$

$v: x \cdot b = \text{if } \langle v: x \cdot b \rangle x \text{ then } x \text{ else null fi}$

$v: A.B \cdot b = (v: A \cdot b), (v: B \cdot b)$

$\{ i: \text{int} \cdot i^2=4 \} = \{-2, 2\}$

$\{ n: \text{nat} \cdot n<3 \} = \{0..3\}$
An expression talks about its nonlocal variables.

\[ \exists n: \text{nat} \cdot x = 2 \times n \]

says

“\( x \) is an even natural”