Big Numbers

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Mathematics and science have given us both the need and the means to represent large numbers. The age of the universe is about 13.77 billion Earth-years; there are about 5×10^{22} atoms in one gram of carbon$_{12}$; the number of stars in the universe is about 7×10^{22}; the diameter of the universe is about 8.8×10^{26} meters.

Mathematically, we can define much larger numbers. Perhaps the simplest example is an exponentiation tower. Define

$$tower\ 0 = 1$$
$$tower\ (n+1) = 2^{(tower\ n)}$$

Here are the first few values:

$$tower\ 0 = 1$$
$$tower\ 1 = 2^{(tower\ 0)} = 2^1 = 2$$
$$tower\ 2 = 2^{(tower\ 1)} = 2^2 = 4$$
$$tower\ 3 = 2^{(tower\ 2)} = 2^4 = 16$$
$$tower\ 4 = 2^{(tower\ 3)} = 2^{16} = 65536$$
$$tower\ 5 = 2^{(tower\ 4)} = 2^{65536} = \text{(a number with about 20,000 digits)}$$

We can say something about the size of *tower* 5 : that it has about 20,000 digits in it. After that, for *tower* 6 and *tower* 7 and so on, there's no easy way to characterize the size of the numbers.

Evolution has produced people who are somewhat adapted to live in small groups, located within a small area somewhere on the surface of planet Earth. As part of that adaptation, we have a good understanding of small natural numbers, from 0 up to a few hundred. After that, for most people, numbers are just a blur. Our past evolution has not given us the ability to understand big numbers. A million, a billion, a trillion, they all seem about the same: unimaginably big.

The largest Roman Numeral is M, standing for a thousand; apparently, in Roman times, there was no need for numbers bigger than a few thousand. When the mayor of Toronto in 2000, Mel Lastman, a man not known for his intelligence, attended a conference in Los Angeles on city transit, he bragged that the TTC (Toronto Transit Commission) carries a million passengers a year. Actually, the TTC carries a million passengers a day, but that seemed the same to him.

In Wanaka, New Zealand, as a millennial project, they laid a line of bricks, one brick representing one year, going back two thousand years. On these bricks they inscribed the names of people and events that were current in that year anywhere in the world. A stroll along that path is a great way to correlate events and periods in different parts of the world, and to get a feel for the relative time spans during and between major events.

The Wanaka path works brilliantly for the two thousand years it represents. But for the age of the universe, each of the two thousand bricks would have to represent 6.8 million years. The existence of humankind would occupy the last quarter of the last brick, with no space to inscribe any of the events that humans are familiar with; most of the rest would be empty, with just the formation of our sun and planets to interest the average person. All of recorded human history would occupy 0.00058 of a brick.

Sometimes natural history museums like to scale the history of the universe into a 24-hour timespan. On that scale, humankind occupies the last 12 seconds of the day. Recorded human history occupies the last 0.025 of a second. Both the Wanaka bricks and the 24-hour clock are great ways to start to understand the age of the universe.

To try to understand astronomical distances, we might represent the diameter of the universe as the length of a football field. Then the diameter of Earth is 0.00000001 times the width of an atom. But since the width of an atom, and the number 0.00000001, are not familiar to most people, this attempt to relate universal distances to human experience fails. If the universe is scaled to a football field, even the diameter of our entire solar system (sun and planets) is about a hundredth of the width of an atom. If we build a model of our solar system using an orange for the Earth, then the diameter of Neptune's orbit is 70 km. Any physical model or drawing of our solar system must be grossly out of scale. If we shrink the Earth to a grain of sand, the universe remains unimaginably big. There is no way to make universal sizes understandable.

In 1957, the Dutch educator Kees Boeke wrote a book named *Cosmic View* in which he used a logarithmic scale to represent astronomical distances. Based on that idea, in 1977, Charles and Ray Eames made a movie named *Powers of Ten*, which zoomed in and out from atoms to the universe. Each zoom in or out increases or decreases the scale by 10 times. From the diameter of an atom (10^{-10} m) to the diameter of the universe (10^{27} m) is 37 zooms. Just 37 zooms is easy to understand. But it is incredibly misleading. The scale that represents distances proportionally is linear; the scale used in *Powers of Ten* is logarithmic. The majority of people do not know the difference between a linear scale and a logarithmic scale; one zoom seems about the same as another. The zoom out from 10^0 m to 10^1 m adds 9 meters to your field of view; the zoom out from 10^{26} m to 10^{27} m adds 900,000,000,000,000,000,000,000,000 meters. Even that sentence is inadequate to explain the mismatch because the length of a number (number of digits; in the previous sentence, that's a 27 digit number) is logarithmic in the size of the number represented. Just to write down that same number using a linear (proportional) representation would take us well beyond our cluster of galaxies.

In Star Wars movies and similar science fiction, people zip around the universe quickly and easily. The stories may be great, but they present a very false sense of universal distances, reinforcing the misconception that all big numbers are the same.

Does it matter whether the general population understands big numbers? For democracy to work, people have to understand how their tax money is spent. In some countries, that's a budget of trillions of the monetary unit, so the amounts are outside the understanding of most people.

Some people don't believe in evolution. They can't see it happening. Telling them that it took 4.3 billion years is just meaningless to them. They believe Earth is about 5000 years old; that's as big a number as they can comprehend. In that short time, there's no way the complexity of life forms around them could have evolved, so someone (God) must have designed it.

If a person is just a thing, how can a person be conscious? To explain how consciousness arises from the complexity of a person's 37 trillion cells is meaningless to most people. So consciousness must be the result of a magical, invisible soul. Part of the appeal of religion over science is that science's explanations involve big numbers that ordinary people cannot understand, and religion's explanations don't.

So yes, it matters whether the general population understands big numbers. And they don't.

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