0[6] Let \( x \) and \( y \) be integer variables. What is the exact precondition for \( x' = y \) to be refined by \( y := 1 \)?

\[
\forall x', y'. x' = y \iff (y := 1)
\]

expand assignment

\[
\forall x', y'. x' = y \iff x' = x \land y' = 1
\]

one-point twice

\[
x = y
\]

1[12] Let \( a \), \( b \), and \( c \) be integer variables. Express

\[
c := a - b - c. \quad b := a - b - c. \quad a := a - b - c. \quad c := a + b + c
\]
as simply as possible without using quantifiers, assignments, or dependent compositions.

\[
c := a - b - c. \quad b := a - b - c. \quad a := a - b - c. \quad a' = a \land b' = b \land c' = a + b + c
\]
Substitution Law

\[
c := a - b - c. \quad b := a - b - c. \quad a' = a - b - c \land b' = b \land c' = (a - b - c) + b + c
\]
arithmetic

\[
c := a - b - c. \quad b := a - b - c. \quad a := a - b - c \land b' = b \land c' = a
\]
Substitution Law

\[
c := a - b - c. \quad a' = a - (a - b - c) - c \land b' = a - b - c \land c' = a
\]
arithmetic

\[
a' = b \land b' = a - b - c \land c' = a
\]
Substitution Law

\[
a' = b \land b' = c \land c' = a
\]

2[15] Let \( x \) be an integer variable and let \( t \) be time. Prove the refinement

\[
P \iff \text{if } x = 0 \text{ then } ok \text{ else } x := x - 1. \quad t := t + 1. \quad P\]

where \( P = \text{if } x \geq 0 \text{ then } t' = t + x \text{ else } t' = \infty \)

\[
\text{By Cases. First case:}
\]

\[
\]

expand \( P \) and \( ok \)

\[
\]

context

\[
\]

reflexive, case base

\[
\]

base

Last case:

\[
\]

expand \( P \)

\[
\]

substitution twice

\[
\]

simplify

\[
\]

context: \( x \geq 0 \Rightarrow (x > 0 \iff x \geq 0) \)

\[
\]

specialization

\[
\Rightarrow P
\]

3 Let \( A \) be an array, and let \( i \) be a variable whose value is an index of \( A \).

(a)[6] Express \( A(A(A \, i)) := A \, i \) without using \( := \).

\[
A' = A(A \, i) \rightarrow A \, i \ | A \land i' = i
\]
or

\[
A'(A(A \, i)) = A \, i \land (\forall j \, j + A(A \, i) \Rightarrow A' \, j = A \, j) \land i' = i
\]
(b)[3] Suppose initially \( A = [2; 0; 1] \) and \( i = 0 \). What are their final values after execution of the array element assignment in part (a)?

\[
A' = [2; 2; 1] \text{ and } i' = 0
\]