

ECE750T30 Test 0 Solutions 2018 October 3

0[9] Using the proof format and laws in the textbook, prove
 $a = (b \Rightarrow a) = a \vee b$

§ $(a = (b \Rightarrow a) = a \vee b)$ symmetry of =
 = $((b \Rightarrow a) = a = a \vee b)$ associativity of =
 = $((b \Rightarrow a) = a = (a \vee b))$ symmetry of = and \vee
 = $((b \Rightarrow a) = (b \vee a) = a)$ inclusion
 = \top

1[9] Let i and j be indexes of list L . Simplify
 $L[0;..min\ i\ j ; max\ i\ j ; min\ i\ j + 1;..max\ i\ j ; min\ i\ j ; max\ i\ j + 1;..#L]$
 Hint: use $\rightarrow |$

§ $i \rightarrow Lj \mid j \rightarrow Li \mid L$

2 There is a saying: “No news is good news.” Let a news story be a text, and let $news$ be all the news stories, and let $good$ be a predicate over $news$. Formalize the saying as a binary expression, assuming it means the same as

(a)[3] “There's no such thing as good news.”

§ $\neg \exists n: news \cdot good\ n$

(b)[3] “The fact that there isn't any news is a good news story.”

§ $“news=null”: news \wedge good\ “news=null”$

(c)[3] “If there isn't any news then that will be a good news story.”

§ $news=null \Rightarrow “news=null”: news \wedge good\ “news=null”$

If “ $news=null$ ”: $news$ then $news=null$ is false, so “ $news=null$ ” is false news, but there's no logical inconsistency.

3[6] Let p be a predicate with domain nat . Express formally that there is at least one and at most a finite number of naturals satisfying predicate p .

§ $1 \leq \phi(\{n: nat \cdot pn\}) < \infty$

or

$1 \leq \phi(\{p\}) < \infty$

4[9] Formalize and disprove the statement “There is a natural number that is not equal to any natural number.”.

§ $\exists n: nat \cdot \forall m: nat \cdot n \neq m$

Leaving the domain nat implicit,

$\exists n \cdot \forall m \cdot n \neq m$

$\Rightarrow \exists n \cdot n \neq n$

$= \exists n \cdot \perp$

$= \perp$

specialization: replace m with n

\neq is irreflexive

Identity