0[9] Using the proof format and laws in the textbook, prove

\[ a = (b \Rightarrow a) \iff a \lor b \]

\[ \begin{align*}
& (a = (b \Rightarrow a) \iff a \lor b) \\
\quad = & ((b \Rightarrow a) = a \iff a \lor b) \quad \text{symmetry of } = \\
\quad = & ((b \Rightarrow a) = a = (a \lor b)) \quad \text{associativity of } = \\
\quad = & ((b \Rightarrow a) = (b \lor a) = a) \quad \text{symmetry of } = \text{ and } \lor \\
\quad = & T
\end{align*} \]

1[9] Let \( i \) and \( j \) be indexes of list \( L \). Simplify

\[ L[0..\min i j ; \max i j ; \min i j + 1..\max i j ; \min i j ; \max i j + 1..#L] \]

Hint: use \( \rightarrow \mid \)

\[ i \rightarrow Lj \mid j \rightarrow Li \mid L \]

2 There is a saying: “No news is good news.”. Let a news story be a text, and let \( news \) be all the news stories, and let \( good \) be a predicate over \( news \). Formalize the saying as a binary expression, assuming it means the same as

(a)[3] “There's no such thing as good news.”

\[ \neg \exists n: news \cdot good n \]

(b)[3] “The fact that there isn't any news is a good news story.”

\[ “news=null”: news \land good “news=null” \]

(c)[3] “If there isn't any news then that will be a good news story.”

\[ news=null \Rightarrow “news=null”: news \land good “news=null” \]

If “news=null”: news then news=null is false, so “news=null” is false news, but there's no logical inconsistency.

3[6] Let \( p \) be a predicate with domain \( nat \). Express formally that there is at least one and at most a finite number of naturals satisfying predicate \( p \).

\[ 1 \leq \phi(\forall n: nat \cdot pn) < \infty \]

or

\[ 1 \leq \phi(\exists p) < \infty \]

4[9] Formalize and disprove the statement “There is a natural number that is not equal to any natural number.”.

\[ \exists n: nat \forall m: nat \cdot n \neq m \]

Leaving the domain \( nat \) implicit,

\[ \exists n: nat \forall m: nat \cdot n \neq m \]

specialization: replace \( m \) with \( n \)

\[ \Rightarrow \exists n: n \neq n \]

\[ \Rightarrow \exists n: \bot \]

\[ \Rightarrow \bot \]

\[ \bot \]

\[ \bot \]