Let $x$ be an integer state variable.

(a) For what exact precondition does $x := x + 1$ move $x$ farther from zero, staying on the same side of zero (if $x$ was zero, then either side will do)?

\[
\begin{align*}
\forall x'. \ x' < x < 0 \lor x' > x > 0 & \iff (x := x + 1) \\
\forall x'. \ x' < x < 0 \lor x' > x + 1 \lor x > x > 0 & \iff x' = x + 1
\end{align*}
\]

(b) For what exact postcondition does $x := x + 1$ move $x$ farther from zero, staying on the same side of zero (if $x$ was zero, then either side will do)?

\[
\begin{align*}
\forall x. \ x < x < 0 \lor x' + x = 0 \lor x' > x > 0 & \iff (x := x + 1) \\
\forall x. \ x' < x < 0 \lor x' > x + 1 \lor x > x > 0 & \iff x' = x + 1
\end{align*}
\]

Let $x$ and $y$ be binary variables. Simplify

\[
\begin{align*}
x := x = y . \ x := x = y & \iff x := x = y. \ x := x = y & \text{Substitution Law} \\
x := x = y . \ x' = (x = y) \land y' = y & \iff x' = (x = y) \land y' = y & \text{Associative Law for binary } = \\
x' = (x = (y = y)) \land y' = y & \iff x' = (x = (y = y)) \land y' = y & \text{Reflexive and Identity laws for } = \\
x := x \land y' = y & \iff x = x \land y' = y & \text{Simplify}
\end{align*}
\]

Let the state variables be $x$, $y$, and $z$. Rewrite \texttt{frame} $x \cdot T$ without using \texttt{frame}.

Say in words what the final value of $x$ is. (Note: $T$ is the “true” binary value.)

\[
\text{frame} x \cdot T \iff y' = y \land z' = z
\]

The final value of $x$ is an arbitrary (or unknown) value of its type.