0[6] Using the proof format and laws in the textbook, prove
\[(a \land b) \lor (b \land c) \lor (a \land c) \equiv (a \lor b) \land (b \lor c) \land (a \lor c)\]

\[\text{§} \quad (a \land b) \lor (b \land c) \lor (a \land c) \quad \text{distribute}\]
\[= (a \lor b) \land (a \lor c) \land (a \lor c) \land (b \lor b) \land (b \lor c) \land (a \lor c) \land (b \lor c) \land (a \lor c) \quad \text{symmetry and idempotence}\]
\[= (a \lor b) \land (a \lor b) \land (a \lor c) \land (b \lor c) \land (a \lor c) \quad \text{absorption}\]
\[= (a \lor b) \land (b \lor c) \land (a \lor c) \]

1[6] A sign says:

<table>
<thead>
<tr>
<th>NO PARKING</th>
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</thead>
<tbody>
<tr>
<td>7-9am</td>
</tr>
<tr>
<td>4-6pm</td>
</tr>
<tr>
<td>Mon-Fri</td>
</tr>
</tbody>
</table>

Using variable \(t\) for time of day and \(d\) for day of week, and time-of-day constants like 7am and 6pm, and day-of-week constants like Mon and Fri, write a binary expression that says when there is no parking.

\[\text{§} \quad \text{My first attempt is}\]
\[(7\text{am} \leq t < 9\text{am} \lor 4\text{pm} \leq t < 6\text{pm}) \land \text{Mon} \leq d \leq \text{Fri}\]

For the time \(t\), it is not important whether we use \(<\) or \(\leq\). But the problem is that the days cycle. Saturday comes after the preceding Monday and before the following Friday. Likewise the times of day cycle, so that midnight comes after the preceding 4pm and before the following 6pm. So I will represent a day as a string of length 3:
\[\text{year; week; day}\]

and a time as a string of length 4:
\[\text{year; week; day; time}\]

I need the weeks to start on a Saturday or a Sunday or a Monday; let’s say Monday. Since a year may not start on Monday, number the first partial week 0, and after that number the weeks starting on Mondays. My next attempt is
\[y; w; \text{Mon} \leq y; w; d \leq y; w; \text{Fri} \]
\[\land \quad (y; w; d; 7\text{am} \leq y; w; d; t < y; w; d; 9\text{am}) \lor (y; w; d; 4\text{pm} \leq y; w; d; t < y; w; d; 6\text{pm})\]

In any given year \(y\) and week \(w\), if the day \(d\) is between Mon and Fri, and on that day the time \(t\) is between 7am and 9am or between 4pm and 6pm, then there is no parking. That’s better than the previous attempt, but it is still wrong concerning the first and last partial weeks in a year.

2[9] A prime number is a natural number greater than 1 whose only factors are 1 and the number itself. A composite number is a natural number greater than 1 with 2 or more (not necessarily distinct) prime factors. Formally express the composite numbers as simply as you can.

\[\text{§} \quad (\text{nat}+2) \times (\text{nat}+2)\]
Express formally that list $L$ is a sorted segment of list $M$. A segment is a consecutive subsequence.

$\exists i, j : \text{nat} \cdot 0 \leq i \leq j \leq \#M \land L = M[i..j] \land \forall k, l : \text{nat} \cdot i \leq k \leq l < j \Rightarrow M_k \leq M_l$

or

$\exists i, j : \text{nat} \cdot 0 \leq i \leq j \leq \#M \land L = M[i..j] \land \forall k, l : \text{nat} \cdot 0 \leq k \leq l < \#L \Rightarrow L_k \leq L_l$

We can express “there is a smallest natural number” as follows:

$\exists n : \text{nat} \land \forall m : \text{nat} \cdot n \leq m$

(a) Now how do we say “Denote that smallest natural number 0.” formally? In other words, how do we say “Let’s call that smallest natural number 0.” formally?

$b : \text{nat} \land \forall m : \text{nat} \cdot b \leq m$

(b) Prove that there are not two different natural numbers that are tied for smallest.

Let $a$ and $b$ be smallest natural numbers.

$b : \text{nat} \land (\forall m : \text{nat} \cdot b \leq m)$

Specialize the first $\forall$ with $b$ for $m$ and specialize the last $\forall$ with $a$ for $m$.

$a \leq b$ \land b \leq a$

Now, from a generic law (antisymmetry) we have

$a = b$