Exam Reminders:
• State your name and student number on the top of this page and on the front of the answer booklet. All answers are written in the answer booklet, not on the question page.
• Do not begin writing the exam until the announcements have ended and the exam facilitator has started the exam.
• As a student, you help create a fair and inclusive writing environment. If you possess an unauthorized aid during an exam, you may be charged with an academic offense.
• Turn off and place all cell phones, smart watches, electronic devices, and unauthorized study materials in your bag under your desk. If it is left in your pocket, it may be an academic offense.
• When you are done your exam, raise your hand for someone to come and collect your exam. Do not collect your bag and jacket before your exam is handed in.
• If you are feeling ill and unable to finish your exam, please bring it to the attention of an exam facilitator so it can be recorded before leaving the exam hall.
• In the event of a fire alarm, do not check your cell phone when escorted outside.

Exam Format and Grading Scheme:
There is 1 question page, 6 questions, and 100 marks.
The value of each question is indicated in square brackets.
A blank answer is worth about one-third of the marks; to that, marks will be added for readable and relevant and correct information, and marks will be subtracted for unreadable or irrelevant or incorrect information.

Students must hand in all examination materials at the end.
1[9] Let $x$ and $y$ be binary variables. Simplify

$$x := x \land y, \quad x := x \land y$$

2 In natural variables $n$ and $s$

(a)[15] prove the refinement

$$s := \Sigma[0..n] \iff \text{if } n=0 \text{ then } s := 0 \text{ else } n := n-1. \quad s := \Sigma[0..n], \quad s := s + n, \quad n := n + 1 \text{ fi}$$

(b)[15] add recursive time and prove a finite time bound.

3[18] Write a program to find the maximum item in a list $L$ of numbers, reporting the answer as the final value of number variable $m$. Write all specifications and refinements formally, but you do not need to prove the refinements. Include recursive time $t$ in your specifications and refinements.

4[13] Here is a nest of loops. All exits are shown. What refinements need to be proven in order to prove that this nest of loops refines specification $S$?

```
do A.
do B.
do C.
exit 1 when u.
exit 2 when v.
exit 3 when w.
D od.
E od.
F od.
```

5 Let $n$ be a natural number. From the construction axiom for $\text{few}$

$$0, n \rightarrow \text{few}: \text{few}$$

(a)[3] what elements are constructed?

(b)[3] give three solutions (considering $\text{few}$ as the unknown).

(c)[3] give the corresponding induction axiom.

(d)[3] state which solution is specified by construction and induction.

6[18] Prove that execution of the program

```
chan c: int c? c! 5
```

deadlocks. Allow 1 time unit for message transmission from output to input. There are no program variables.