LAST (FAMILY) NAME_____

FIRST (GIVEN) NAME_____

STUDENT NUMBER_____

UNIVERSITY OF TORONTO
Faculty of Arts & Science

DECEMBER 2022 EXAMINATIONS

CSC465H1F and CSC2104H1F
Formal Methods of Software Design

Duration: 2 hours

Aids allowed: one letter-sized page, both sides
and the laws from the textbook, 14 pages

Exam Reminders:

- State your name and student number on the top of this page and on the front of the answer booklet. All answers are written in the answer booklet, not on the question page.
- Do not begin writing the exam until the announcements have ended and the exam facilitator has started the exam.
- As a student, you help create a fair and inclusive writing environment. If you possess an unauthorized aid during an exam, you may be charged with an academic offense.
- Turn off and place all cell phones, smart watches, electronic devices, and unauthorized study materials in your bag under your desk. If it is left in your pocket, it may be an academic offense.
- When you are done your exam, raise your hand for someone to come and collect your exam. Do not collect your bag and jacket before your exam is handed in.
- If you are feeling ill and unable to finish your exam, please bring it to the attention of an exam facilitator so it can be recorded before leaving the exam hall.
- In the event of a fire alarm, do not check your cell phone when escorted outside.

Exam Format and Grading Scheme:
There is 1 question page, 7 questions, and 100 marks.
The value of each question is indicated in square brackets.
A blank answer is worth about one-third of the marks;
to that, marks will be added for readable and relevant and correct information,
and marks will be subtracted for unreadable or irrelevant or incorrect information.

Students must hand in all examination materials at the end.

start of exam

1 Let a , b , and c be integer variables. Express each of the following as simply as possible without using quantifiers, assignments, sequential compositions, or concurrent compositions. Ignore time.

(a)[9] $a := a-b.\ b := a-b.\ a := a+b$

(b)[9] $a := a-b.\ (a := a+b \parallel b := a+b)$

2[9] Let the variables be x, y: *int* . Write a program to refine specification $\neg(x := y)$. Prove your refinement.

3[21] Let x be an integer variable, and let t be the time variable. Find an implementable time specification P such that

$P \Leftarrow$ **if** $x=0$ **then** *ok* **else** $x := x+1.\ t := t+1.\ P$ **fi**

and prove the refinement.

4[21] Let n be a natural constant, and let f and i be natural state variables. Using the proof methods of this course, prove that the program

$f := 1.\ i := 0.$ **while** $i<n$ **do** $i := i+1.\ f := f \times i$ **od**

refines the specification $f' = n!$ where

$n! = \Pi i: 0,..n \cdot i+1 = 1 \times 2 \times 3 \times ... \times n$

5[9] Using the methods of this course, prove $a \times x^2 + b \times x + c = 0$ is an invariant for $(x := a \times x + b.\ x := -x/a)$ where the only variable is x .

6 Here is a construction axiom for *what* .

2, *what*+*what*, *what*×*what*: *what*

(a)[3] What is the induction axiom?

(b)[9] What are the first four iterations of recursive construction?

(c)[3] Guess what *what* is.

7[7] Here is a program to print the sequence of natural numbers, one per time unit, using recursive time.

$P \Leftarrow m := 0.\ Q$

$Q \Leftarrow c!\, m.\ m := m+1.\ t := t+1.\ Q$

Define reasonable specifications P and Q , including messages values and times. You do not need to prove the refinements.

end of exam