CSC465/2104 Test 2 2022 November 17, 4:10pm-5pm

1 page, 3 questions, 42 marks, 50 minutes
Aids allowed: one letter-sized page, both sides
and the laws from the textbook, 14 pages

The value of each question is indicated in square brackets.
A blank answer is worth about one-third of the marks;
to that, marks will be added for readable and relevant and correct information,
and marks will be subtracted for unreadable or irrelevant or incorrect information.

0[18] Let n be a natural variable. You are given the refinement

$$P \;\Leftarrow\; \textbf{if } n{=}0 \textbf{ then } n'{>}0.\ P \textbf{ else } ok \textbf{ fi}$$

Using recursive construction, find a solution for P. You may ignore time.

1[12] Here is data-queue theory.

(0) $emptyq\colon queue$
(1) $join\ q\ x\colon queue$
(2) $join\ q\ x \ne emptyq$
(3) $join\ q\ x = join\ r\ y \;=\; q{=}r \land x{=}y$
(4) $emptyq, join\ B\ X\colon B \;\Rightarrow\; queue\colon B$
(5) $leave\ (join\ emptyq\ x) \;=\; emptyq$
(6) $q{\ne}emptyq \;\Rightarrow\; leave\ (join\ q\ x) = join\ (leave\ q)\ x$
(7) $front\ (join\ emptyq\ x) \;=\; x$
(8) $q{\ne}emptyq \;\Rightarrow\; front\ (join\ q\ x) = front\ q$

Prove that if you start with an empty queue, and join two items, the first item joined is the front of the queue.

2 Implementer's variables $p, q\colon real$ represent two points along a line. Each number tells the distance of one point from the origin (a standard point). They must be reimplemented by one implementer's variable $r\colon real$ which tells the distance from p to q. For examples, if $p{=}3$ and $q{=}5$, then $r{=}2$; if $p{=}5$ and $q{=}3$, then $r{=}{-}2$.

(a)[3] What is the data transformer? (no proof)

(b)[9] A user has binary variable b and operation

$$compare \;=\; b{:=}\ q{\ge}p$$

Use your transformer from part (a) to transform operation $compare$.