0[9] Let \( a \) and \( b \) be binary variables. Using the proof format and laws in the textbook, prove
\[
a = (b \implies a) \equiv a \lor b
\]

\[
\begin{align*}
(a = (b \implies a) & \equiv a \lor b) \\
= & ((b \implies a) = a \equiv a \lor b) \\
= & ((b \implies a) \equiv a = (a \lor b)) \\
= & ((b \implies a) = (b \lor a) = a) \\
= & T
\end{align*}
\]

1[9] Let \( i \) and \( j \) be indexes of list \( L \). Simplify
\[
L[0;..\min i j ; \max i j ; \min i j + 1;..\max i j ; \min i j ; \max i j + 1;..#L]
\]
Hint: use \( \rightarrow | \)
\[
\begin{align*}
i \rightarrow Lj & | j \rightarrow Li | L
\end{align*}
\]

2 There is a saying: “No news is good news.”. Let a news story be a text, and let \( news \) be all the news stories, and let \( good \) be a predicate over \( news \). Formalize the saying as a binary expression, assuming it means the same as
(a)[3] “There's no such thing as good news.”
\[
\neg \exists n: news \cdot good n
\]
(b)[3] “The fact that there isn't any news is a good news story.”
\[
\neg \exists n: news \cdot good \ “news=null”
\]
(c)[3] “If there isn't any news then that will be a good news story.”
\[
\neg news=\null \Rightarrow \neg \exists n: news \cdot good \ “news=null”
\]
If “news=\null” \( \land \) news then \( news=\null \) is false, so “news=\null” is false news, but there's no logical inconsistency.

3[6] Let \( p \) be a predicate with domain \( nat \). Express formally that there is at least one and at most a finite number of naturals satisfying predicate \( p \).
\[
\begin{align*}
1 \leq \varepsilon(\exists n: nat \cdot pn) & < \infty \\
or \quad 1 \leq \varepsilon(\exists p) & < \infty
\end{align*}
\]

4[9] Formalize and disprove the statement “There is a natural number that is not equal to any natural number.”.
\[
\begin{align*}
\exists n: nat \cdot \forall m: nat \cdot n \neq m
\end{align*}
\]
Leaving the domain \( nat \) implicit,
\[
\exists n \cdot \forall m: n \neq m
\]
specialization: replace \( m \) with \( n \)
\[
\Rightarrow \exists n: n \neq n
\]
+ is irreflexive
\[
\equiv \exists n: \bot
\]
Identity
\[
\equiv \bot
\]